

# Donkey anaphora: Farmers and bishops

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## 1 Introduction: First Order Logic (FOL) and pronouns

Pronouns in natural languages and variables in formal languages are context sensitive to the extreme, and this dependence is most transparent when they are presented out of context. *But then he was* is the opening sentence of a chapter in Capote's *In cold blood*, and it surely leaves the reader puzzled as to what the sentence means until she realizes that the pronoun (as well as the past tense and the VP ellipsis) receive its reference from the previous chapter. Similarly, a formula in classical first-order logic (FOL), as in (1), while interpretable, states only that for some value of  $x$  assigned by the contextually-provided assignment function, it is true that  $x$  sleeps. This is very different from the interpretation of the same formula when embedded in the scope of a quantifier, e.g., a universal quantifier (everybody sleeps) or a negated existential quantifier (nobody sleeps).

- (1) SLEEP( $x$ )
- (2) a.  $\forall x(\text{SLEEP}(x))$   
 b.  $\neg\exists x(\text{SLEEP}(x))$

Given that both pronouns and variables are context sensitive, one might wonder whether a natural-language pronoun could be translated as a FOL variable (and vice versa). This intuitively appealing translational equivalence is difficult to formalize for several reasons, all of which can ultimately be traced back to the fact that FOL is too restrictive to capture the range of meanings and meaning interactions exhibited by natural languages. The reader confused by Capote's *But then he was* will surely recover the reference by eventually recalling the end of the previous chapter in the book. But FOL quantifiers erase variable-assignment manipulations after the subformulas in their scope are interpreted, and therefore, variables simply cannot be semantically dependent on FOL quantifiers when they are not in their scope.

The difference between the behavior of FOL variables and natural-language pronouns are commonly demonstrated with two kinds of phenomena: anaphora across clausal conjuncts and donkey anaphora. While the main focus of this chapter is donkey anaphora, it is instructive and easier to start with the first kind of phenomena. The next section discusses both in detail.<sup>1</sup>

## 2 Problems for a compositional semantics based on classical FOL semantics

### 2.1 Anaphora across (Clausal) Conjuncts

The example below shows a simple sentence with two possible antecedents, one introduced by *every* and the other by the indefinite article *a*, and one anaphoric pronoun bound by the first antecedent. To guide the reader towards the intended reading, we superscript antecedents with the variable they introduce and subscript anaphors with the variable they retrieve.

- (3) Every <sup>$x$</sup>  man saw a <sup>$y$</sup>  friend of his <sub>$x$</sub> .

We say that the determiner *every* and the indefinite article *a* are antecedents because they can be straightforwardly translated into FOL as introducing universal and existential quantifiers, and they can therefore bind anaphoric elements in their scope. The schemata in (4) and (5) give typical English-to-FOL translations for sentences in which these quantifiers function as subjects; the corner quotes indicate that the appropriate translations for NPs / VPs should be substituted in the FOL translations. The NP and VP together provide the scope of the FOL quantifiers, as exemplified in Figure 1 for the universal quantifiers.

- (4) *every* NP VP  $\rightsquigarrow \forall x(\ulcorner NP \urcorner(x) \rightarrow \ulcorner VP \urcorner(x))$
- (5) *a* NP VP  $\rightsquigarrow \exists x(\ulcorner NP \urcorner(x) \wedge \ulcorner VP \urcorner(x))$

The FOL translations of *every* and *a* are nothing more than instructions to manipulate variable assignments in their scope. For example,  $\forall x(\varphi)$  is just an instruction to assign all individuals to  $x$  one by one and to check that the scope  $\varphi$  is true with respect to these modified

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<sup>1</sup>For a more detailed and technical discussion of many of the issues discussed in this chapter, see Brasoveanu, (2013) and references therein.

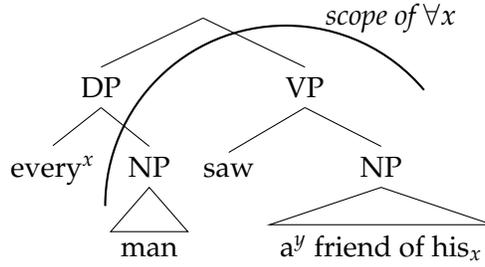


Figure 1: Scope of *every*

assignments. Similarly,  $\exists x(\varphi)$  is an instruction to check that  $\varphi$  is true with respect to an assignment in which  $x$  is assigned some individual from the domain of interpretation. Consequently, any anaphor<sup>2</sup> to a quantifier appearing in its scope can be translated as the variable introduced by the quantifier, and it will receive the same interpretation as the variable the quantifier introduces. Since *his* in (3) is in the scope of *every* (see also Figure 1), it can be treated as a variable and (6) derives the intuitively correct truth conditions (for every man  $x$ ,  $x$  saw a friend of  $x$ ).

$$(6) \quad \forall x(\text{MAN}(x) \rightarrow \exists y(\text{FRIEND-OF}(y, x) \wedge \text{SEE}(x, y)))$$

The situation changes in (7), where *every* scopes only over *Harry Potter book* (the c-commanded NP) and the verb *bought* (the sister of the whole DP). The anaphor  $it_x$  is outside of the scope of *every*. Since the variable assignment is modified with respect to  $x$  *only* in the scope of *every*,  $it_x$  (translated as the variable  $x$ ) cannot be referentially dependent on / bound by *every*. This in fact is a good result because (7) lacks a reading in which *it* is bound by *every*, namely, Mary saw a man such that for every Harry Potter book, he had bought it and had read it in a day.

$$(7) \quad \# \text{ Mary saw } a^y \text{ man who had bought every}^x \text{ Harry Potter book and who had read } it_x \text{ in a day.}$$

Unfortunately, FOL's successful account of *every* leads to an incorrect analysis of indefinites. Example (8) contains an instance of felicitous anaphora between the pronoun  $it_z$  and the indefinite  $a^z$  *Harry Potter book* that is syntactically parallel to the infelicitous anaphora in (7) above.

$$(8) \quad \text{Mary saw } a^y \text{ man who had bought } a^z \text{ Harry Potter book and who had read } it_z \text{ in a day.}$$

The problem is that *it* can refer to the Harry Potter book  $z$  that the man  $y$  had bought even though *it* appears outside the scope of the indefinite  $a$ . More concretely, we can see that the formula in (9) below does not derive the intuitively correct truth conditions for (8) because the variable  $z$  in  $\text{READ}(y, z)$  is not in the scope of  $\exists z$ , hence not influenced by (the interpretation of) this quantifier. Informally, (9) is true iff Mary saw a man who had bought a Harry Potter book and read some contextually salient entity (not necessarily the Harry Potter book just mentioned) – which is not the intended interpretation of (8).

$$(9) \quad \exists y(\text{MAN}(y) \wedge \text{SEE}(m, y) \wedge \underbrace{\exists z (\text{HP\_BOOK}(z) \wedge \text{BUY}(y, z))}_{\text{syntactic and semantic scope of } \exists z}) \wedge \text{READ}(y, z))$$

<sup>2</sup>We are using the term 'anaphor' loosely here: it simply means an expression that is referentially dependent on the quantifier it is coindexed with.

At this point, one might conclude that example (8) show us that anaphors should not be translated as variables. But if they are not variables, what are they?

The first possibility that comes to mind is to translate them in the same way as the antecedent. In that case,  $it_z$  would be the indefinite  $a^z$  *Harry Potter book*, as shown in (10) below. But that also fails to derive the intuitively correct truth conditions: informally, (10) says that Mary saw a man who had bought a Harry Potter book and had read some Harry Potter book or other (not necessarily the one he bought), which is not the interpretation we are after. In particular, the formula is true in a situation in which the man seen by Mary read another Harry Potter book than the one he bought – while sentence (8) is false in such a situation.

$$(10) \quad \exists y(\text{MAN}(y) \wedge \text{SEE}(m, y) \wedge \exists z(\text{HP\_BOOK}(z) \wedge \text{BUY}(y, z)) \wedge \exists z(\text{HP\_BOOK}(z) \wedge \text{READ}(y, z)))$$

What we actually need is to extend the scope of the existential quantifier to allow it to bind the variable contributed by the pronoun. In other words, the scope of the indefinite should include the complete second conjunct *who had read it<sub>z</sub> in a day*, as shown in Figure 2.

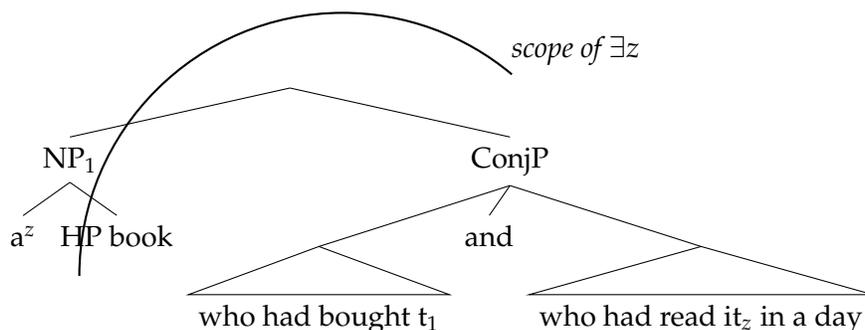


Figure 2: Desired representation of the relative clauses in (8)

This representation correctly lets the existential quantifier bind the anaphor – see the formula in (11). However, it requires a non-trivial step: extending the scope of  $\exists z$  beyond the clause and the conjunct in which it appears.

$$(11) \quad \exists y(\text{MAN}(y) \wedge \text{SEE}(m, y) \wedge \exists z \quad (\text{HP\_BOOK}(z) \wedge \text{HAVE}(y, z) \wedge \text{READ}(y, z)) \quad )$$

This would require the indefinite *a Harry Potter book* to be displaced from its original position in the first conjunct (*who had bought a Harry Potter book*) so that it scopes over the whole coordination. Such a displacement violates syntactic constraints, in particular the Coordinate Structure Constraint which bans the displacement of an expression from only one of two conjuncts (see Ross 1967).

But more importantly, allowing this kind of scope extension makes it hard to account for the infelicity of (7) (under the intended reading): if an existential quantifier can extend its scopal domain to antecede pronouns, why can't universals do the same? A possible response to this is that indefinites enjoy freer scope than *bona fide* quantifiers (Farkas, 1981; Fodor and Sag, 1982, a.o.). It is therefore important to show that the behavior of anaphora is not a sub-problem of the scopal behavior problem posed by indefinites (free scope) vs. universals (restricted scope).

First, while allowing indefinites to take exceptionally wide scope in a sentence could explain their wide scope in clausal conjuncts, (12a), it would not explain why indefinites can antecede anaphora across sentences, (12b).

- (12) a.  $A^x$  man had  $a^y$  mustache and  $it_y$  was black.  
 b.  $A^x$  man had  $a^y$  mustache.  $It_y$  was black.

Quantifier scope is a sentence-internal phenomenon and as far as we know, there is no evidence that quantifiers in general can extend their scope beyond their own sentence.

Antecedents other than indefinites would also cause problems, as shown by the example below from Evans, 1977 in which a downward entailing quantifier is anaphorically picked up across a clausal boundary.

- (13)  $Few^x$  congressmen admire Kennedy, and  $they_x$  are very junior.

There are two problems with (13). First, it has been argued that unlike indefinites, modified numerals and non-upward entailing quantifiers do not take exceptionally wide scope (Winter, 1997). Second, allowing *few congressmen* to take scope outside of the first clausal conjunct to bind the anaphoric pronoun in the second conjunct will not yield the correct interpretation: it would state that few congressmen admire Kennedy and are very junior. This interpretation would be true if many congressmen admire Kennedy, as long as only few of them are very junior. But intuitively, (13) is not true in this case. Thus, allowing (some) quantifiers to take exceptionally wide scope is not enough to deal with anaphora across conjuncts and sentences.

## 2.2 Donkey Anaphora

Donkey anaphora was brought to the attention of philosophers of language and linguists in Geach, (1962) (even though Geach himself did not use this term for his examples). There are two main requirements a sentence should satisfy to qualify as an instance of donkey anaphora:

- i. The sentence should be of the form  $Q(\dots NP^x \dots)(\dots it_x / them_x \dots)$ , where:
  - $Q$  is a (nominal or adverbial) quantifier (*every, most, always, often* etc.)
  - the first pair of brackets after  $Q$  marks the restrictor and the second pair marks the nuclear scope of  $Q$
  - $NP^x$  is an indefinite
  - the pronoun  $it_x / them_x$  has the indefinite as its antecedent, as indicated by its coindexation with the indefinite NP
- ii. The indefinite (and consequently, the pronoun) co-varies with  $Q$

A classic example of donkey anaphora is provided in (14), where:<sup>3</sup>

- $Q = every$
- restrictor = *farmer who owns a<sup>y</sup> donkey*; nuclear scope = *beats it<sub>y</sub>*

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<sup>3</sup>Geach, 1962 used *any* rather than *every* as the quantifier  $Q$ . *Any* has been substituted by *every* in typical examples of donkey anaphora cited in the formal semantics literature, presumably because the interpretation of *any* introduces semantic complications that are orthogonal to the matters at hand.

- NP =  $a^y$  donkey

(14) Every<sup>x</sup> farmer who owns a<sup>y</sup> donkey beats it<sub>y</sub>.

Another classic example of donkey anaphora is provided in (15). In this case,  $\mathcal{Q}$  is assumed to be (something like) a silent adverb of quantification corresponding roughly to *always*.

(15) If a<sup>x</sup> farmer owns a<sup>y</sup> donkey, he<sub>x</sub> beats it<sub>y</sub>.

We will show that we cannot use FOL to compositionally derive the correct meaning of (14), and it would be straightforward to do the same for (15). Using the translation schemata in the previous section, we compositionally derive the translation in (16). But this is not the correct interpretation; the problem is, yet again, one of scope: the existential quantifier in the restrictor does not have all the instances of the variable  $y$  in its scope.

(16)  $\forall x((\text{FARMER}(x) \wedge \exists y \underbrace{(\text{DONKEY}(y) \wedge \text{OWN}(x, y))}_{\text{syntactic and semantic scope of } \exists y}) \rightarrow \text{BEAT}(x, y))$

The correct FOL translation of sentence (14) is provided in (17), which could be paraphrased as ‘for every farmer and every donkey, if the farmer owns the donkey *s/he* beats it’. This translation, however, is not compositionally derived from (14): there is no subformula in (17) that corresponds to the indefinite *a<sup>y</sup> donkey* in (14) or to the entire relative clause *who owns a<sup>y</sup> donkey*.

(17)  $\forall x \forall y((\text{FARMER}(x) \wedge \text{DONKEY}(y) \wedge \text{OWN}(x, y)) \rightarrow \text{BEAT}(x, y))$

We noted in the previous subsection that the problematic binding of indefinites across clausal conjuncts could not be solved even if we let indefinites take exceptionally wide scope. The same conclusion can be reached here in a different way, and it is instructive to consider it in detail. Suppose we assign the indefinite *a donkey* wide scope, as shown in Figure 3. This representation would allow the indefinite to bind *it<sub>y</sub>*. But it would yield the interpretation that there is a donkey such that every farmer who owns it beats it. This is a possible, albeit less salient reading of (14). However, this is not the interpretation we are trying to derive: the indefinite (and consequently, the pronoun) should co-vary with  $\mathcal{Q}$ , which is not the case if the indefinite outscopes the quantifier.

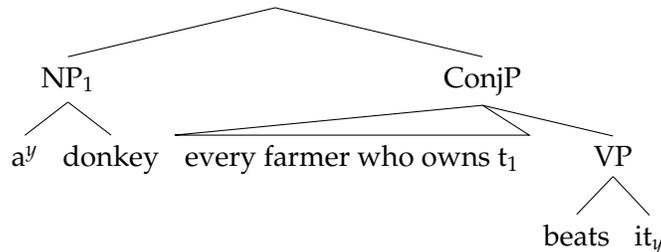


Figure 3: Wide scope of *a donkey* in (14)

### 3 Two approaches to donkey anaphora

Donkey anaphora and anaphora across clauses challenge the connection between language and logic. Two major classes of solutions were developed for this problem in the linguistic literature.

The first class of solutions gives up a uniform analysis of pronouns as variables. Instead, it is assumed that pronouns are disguised definite descriptions. This approach was anticipated in Geach, (1962): although Geach was mainly concerned with the analysis of pronouns that corresponded to variables (in his view), he also discussed cases in which pronouns could be substituted by their linguistic antecedent without any change in the resulting meaning of the sentence. He dubbed such cases ‘pronouns of laziness’. One example of pronouns of laziness he considered is *His sudden elevation to the peerage was a surprise to Smith*, where changing *his* into *Smith’s* has no effect other than a stylistic alteration. Pronouns of laziness could be analyzed as disguised definite descriptions, but this analysis cannot be straightforwardly extended to donkey anaphora. We will return to this class of approaches later (Section 8).

For now our focus will be on the second class of solutions that retain the assumption that pronouns should be uniformly treated as variables. We can maintain this assumption *and* compositionally derive the most salient interpretation for donkey anaphora if we switch from a static to a dynamic semantics for first-order logic (Kamp, 1981; Heim, 1982; Groenendijk and Stokhof, 1991; Kamp and Reyle, 1993, a.o.). Dynamic semantics captures the fact that interpretation is context-sensitive, just like static semantics, but also accounts for the fact that the interpretation context relative to which an expression is semantically evaluated is changed / updated as a side effect of interpreting previous expressions. This means, in particular, that dynamic semantics relinquishes the limiting assumption of classical FOL that quantifiers erase variable-assignment manipulations after the subformulas in their scope are interpreted.

### 4 A solution to donkey problems: dynamic semantics

The main idea behind dynamic semantics is that propositions are evaluated relative to a *pair* of variable assignments rather than a single assignment. That is, propositions express relations between variable assignments. We will use  $g, h, k, g', h', k', \dots$  for variable assignments and  $\varphi, \psi, \chi, \dots$  for formulas. The notation  $\llbracket \varphi \rrbracket^{(g,h)}$  – or equivalently  $g \llbracket \varphi \rrbracket h$  – indicates that the proposition denoted by  $\varphi$  is true relative to the input variable assignment  $g$  and the output variable assignment  $h$ . That is, the denotation of  $\varphi$ , symbolized as  $\llbracket \varphi \rrbracket$ , is a non-deterministic update over variable assignments: it takes an input assignment  $g$  (this is interpretation context that the static interpretation function is relativized to), it interprets  $\varphi$  relative to this input context and in the process updates  $g$  to an output context  $h$  that will be used as the input context for subsequent expressions. Specifically,  $\llbracket \varphi \rrbracket$  is a binary relation over variable assignments: an input assignment  $g$  can in principle be updated to a variety of output assignments  $h_1, h_2, \dots$ . This is what we mean by non-deterministic update (a deterministic update would be functional, not relational); this feature is not specific to dynamic semantics – it is directly inherited from the static semantics for existentials that requires *some* witness or other to satisfy the nuclear scope of the existential.

Apart from formulas, we will have to talk about the following elements in our language:

- terms: individual variables  $x, y, z, \dots$  or individual constants  $j, m, \dots$ , where:

- the denotation of individual variables is provided by assignment functions, and they denote individuals in the universe  $g(x), g(y), \dots$
- the denotation of individual constants is provided by the basic interpretation function  $I$  that is part of the model, and they also denote individuals in the universe  $I(j), I(m), \dots$
- one-place relations (DONKEY, FARMER, ...) and two-place relations (OWN, BEAT, ...), whose denotations are provided by  $I$  and are sets of entities and sets of pairs of entities respectively
- quantifiers:  $\forall x, \exists x$
- dynamic conjunction: ;
- dynamic implication:  $\rightarrow$

Atomic formulas are interpreted as shown in (18), where  $P, R$  are metavariables over unary and binary relations and  $t_1, t_2$  are metavariables over terms. Note that the formulas created by combining relations with arguments denote sets of assignment pairs, but the pairs have a special property: the input and output assignments are identical ( $g = h$ ). That is, these simple denotations do not modify assignments in any way, which is why they are called *tests*: their only role is to weed out assignments that do not satisfy predications.

$$(18) \quad \begin{aligned} \text{a. } \llbracket P(t_1) \rrbracket &= \{ \langle g, h \rangle : g = h \text{ and } g/I(t_1) \in I(P) \} \\ \text{b. } \llbracket R(t_1, t_2) \rrbracket &= \{ \langle g, h \rangle : g = h \text{ and } \langle g/I(t_1), g/I(t_2) \rangle \in I(R) \} \end{aligned}$$

Such formulas, then, do not exploit the fact that we operate with pairs of variable assignments since they specify that the relation that holds between assignments is a trivial one (identity). The situation gets more interesting when we consider connectives. Dynamic conjunction, used for the interpretation of conjunctions as well as sequences of sentences, is evaluated as shown in (19).

$$(19) \quad \llbracket \varphi; \psi \rrbracket = \{ \langle g, h \rangle : \text{there exists a } k \text{ such that } g \llbracket \varphi \rrbracket k \text{ and } k \llbracket \psi \rrbracket h \} = \llbracket \varphi \rrbracket \bullet \llbracket \psi \rrbracket$$

Thus, dynamic conjunction ; denotes relation composition  $\bullet$ . To see this, consider an unrelated but more straightforward example of relation composition: GRANDMOTHER-OF is the composition of relations PARENT-OF and MOTHER-OF. We can write this as

$$\begin{aligned} \text{GRANDMOTHER-OF} &= \text{PARENT-OF} \bullet \text{MOTHER-OF}(c, b) \\ &= \{ \langle a, b \rangle : \text{there exists a } c \text{ such that} \\ &\quad \text{PARENT-OF}(a, c) \text{ and MOTHER-OF}(c, b) \}. \end{aligned}$$

To make the parallel between this composition of relations between people and the composition of relations over variable assignments in (19) clearer, consider the following example. Assume we have two relations over variable assignments,  $\mathcal{R}$  and  $\mathcal{R}'$ .  $\mathcal{R}$  relates a pair of assignments for which it holds that they differ from each other at most with respect to the value assigned to  $x$ . The requirement that two assignments  $g$  and  $h$  differ at most with respect to the value they assign to  $x$  is standardly symbolized as ' $h[x]g$ '. Using this notation, we say that  $\mathcal{R} = \{ \langle g, h \rangle : h[x]g \}$ . Now assume the relation  $\mathcal{R}'$  consists of pairs of assignments such that

the input and output assignments are identical and they assign the individual **john** to  $x$ . That is,  $\mathcal{R}' = \{\langle g, h \rangle : g = h \text{ and } h(x) = \mathbf{john}\}$ . What, then, will  $\mathcal{R} \bullet \mathcal{R}'$  be? Applying the definition of relation composition, we get:

$$(20) \quad \begin{aligned} \mathcal{R} \bullet \mathcal{R}' &= \{\langle g, h \rangle : \text{there exists a } k \text{ such that } k[x]g \text{ and } k = h \text{ and } h(x) = j\} \\ &= \{\langle g, h \rangle : h[x]g \text{ and } h(x) = j\} \end{aligned}$$

In words,  $\mathcal{R} \bullet \mathcal{R}'$  relates an input and an output assignment iff they differ from each other at most with respect to the value of  $x$  and the output assignment assigns the individual **john** to  $x$ . The binary relation  $\mathcal{R} \bullet \mathcal{R}'$  happens to be a function in this case (this is obviously not the case in general), namely:  $[\mathcal{R} \bullet \mathcal{R}'](g) = g^{x:=\mathbf{john}}$ , where  $g^{x:=\mathbf{john}}$  is the assignment that is exactly like  $g$  except it assigns **john** to  $x$  (commonly used when providing the semantics of existential quantification in classical, static FOL).

Importantly, relation composition  $\bullet$  is not commutative, so dynamic conjunction  $;$  is not commutative unlike its static counterpart  $\wedge$  (although both dynamic and static conjunction are associative). For example,

$$(21) \quad \begin{aligned} \mathcal{R}' \bullet \mathcal{R} &= \{\langle g, h \rangle : \text{there exists a } k \text{ such that } g = k \text{ and } k(x) = j \text{ and } h[x]k\} \\ &= \{\langle g, h \rangle : g(x) = j \text{ and } h[x]g\}. \end{aligned}$$

That is, the composition  $\mathcal{R}' \bullet \mathcal{R}$  relates an input and an output assignment iff the input assignment assigns **john** to  $x$  and the output assignment different from the input one at most with respect to the value of  $x$ , which can be any individual in the domain.

Understanding dynamic conjunction becomes very important when we consider the interpretation of the existential quantifier, given in (22a). The most important part of the semantic clause for existential quantification is the first conjunct ' $k[x]g$ '. This allows  $g$  to be modified with respect to  $x$ . The second conjunct ensures that the resulting, modified assignment satisfies the scope of the existential quantifier. If we introduce an atomic formula for random assignment  $[x]$ , we can decompose the existential quantification clause in (22) into two distinct formulas that are dynamically conjoined, as shown in (22b) and (22c). The equivalence in (22c) shows that in dynamic semantics, existential quantification has unlimited semantic scope to the right (all else being equal).

$$(22) \quad \begin{aligned} \text{a. } \llbracket \exists x \varphi \rrbracket &= \{\langle g, h \rangle : \text{there exists a } k \text{ such that } k[x]g \text{ and } k \llbracket \varphi \rrbracket h\} \\ \text{b. } \llbracket [x] \rrbracket &= \{\langle g, h \rangle : h[x]g\} \\ \text{c. } \llbracket \exists x \varphi \rrbracket &= \llbracket [x]; \varphi \rrbracket \end{aligned}$$

We keep the translation schema for indefinites as in FOL, but we update it with dynamic operators:

$$(23) \quad \begin{aligned} a \text{ NP VP} &\rightsquigarrow \exists x(\ulcorner NP \urcorner(x); \ulcorner VP \urcorner(x)) \\ &\rightsquigarrow [x]; \ulcorner NP \urcorner(x); \ulcorner VP \urcorner(x) \end{aligned}$$

At this point, our dynamic semantics can already account for anaphora across clausal conjuncts. Consider the following example, repeated from above:

$$(24) \quad A^x \text{ man had a}^y \text{ mustache and it}_y \text{ was black.}$$

This example is interpreted as shown below (the abbreviations  $\varphi$  and  $\psi$  are for presentation purposes only and will be used shortly).

$$(25) \quad \underbrace{\exists x(\text{MAN}(x); \underbrace{\exists y(\text{MUSTACHE}(y); \text{HAVE}(x, y))}_{\psi})}_{\varphi}; \text{BLACK}(y)$$

Let us break down the formula. First, we can see that there are two conjuncts,  $\varphi$  and  $\text{BLACK}(y)$ . That is, the formula is interpreted as shown below (using the definitions in (18) and (19) and simplifying slightly):

$$(26) \quad \llbracket (25) \rrbracket = \llbracket \varphi; \text{BLACK}(y) \rrbracket = \{ \langle g, h \rangle : g \llbracket \varphi \rrbracket h \text{ and } h(y) \in I(\text{BLACK}) \}$$

We can expand  $\varphi$  by decomposing the existential quantifier as shown in (22c). The result is:

$$(27) \quad \llbracket \varphi \rrbracket = \llbracket [x]; \text{MAN}(x); \psi \rrbracket$$

Finally,  $\psi$  can be expanded in a similar way:

$$(28) \quad \llbracket \psi \rrbracket = \llbracket [y]; \text{MUSTACHE}(y); \text{HAVE}(x, y) \rrbracket$$

Putting this all together, we end up with the following final interpretation:

$$(29) \quad \begin{aligned} \llbracket (25) \rrbracket &= \llbracket [x]; \text{MAN}(x); [y]; \text{MUSTACHE}(y); \text{HAVE}(x, y); \text{BLACK}(y) \rrbracket \\ &= \{ \langle g, h \rangle : \text{there exists a } k \text{ such that } k[x]g \text{ and } k(x) \in I(\text{MAN}) \text{ and } h[y]k \text{ and} \\ &\quad h(y) \in I(\text{MUSTACHE}) \text{ and } \langle h(x), h(y) \rangle \in I(\text{HAVE}) \text{ and } h(y) \in I(\text{BLACK}) \} \end{aligned}$$

In words, the formula in (25) denotes the set of all pairs of assignments  $\langle g, h \rangle$  such that:

- $h$  differs from  $g$  at most with respect to the values of  $x$  and  $y$
- $h(x)$  is a man
- $h(y)$  is a mustache that  $h(x)$  has
- $h(y)$  is black

Note that  $h(y)$  has the same value throughout the formula and in particular,  $h(y)$  that has the property of being black is the same  $h(y)$  that is a mustache of  $h(x)$ . This is so even though the syntactic scope of  $\exists y$  is restricted to  $\psi$ . That is, all occurrences of  $y$  are semantically bound by the existential  $\exists y$  even if they are not ‘syntactically’ bound.

This effect is a consequence of the way the whole system is set up, but the most relevant part is dynamic conjunction – in its explicit guise as a sentential operator and in its implicit guise as part of the interpretation rule for dynamic existential quantification. This ensures that after the formula that is syntactically in the scope of the existential quantifier  $\exists y$  is interpreted, the modified variable assignment is not discarded. Instead, it serves as the input for the subsequent formula  $\text{BLACK}(y)$ . (This can be clearly seen in (26) or in (29)). Since  $\text{BLACK}(y)$  does not modify assignment functions in any way, it simply receives whatever input assignment is delivered to it and tests it. In other words, it assigns the same value to  $y$  as the one  $y$  received in the scope of the existential quantifier. The only thing it can do is to discard assignments in which  $y$  is not black, i.e., remove assignments in which  $y$  is in fact  $x$ ’s mustache but of a different color.

We can summarize our findings about (25) with the following equivalence, which holds in the presented dynamic system without the usual static FOL restriction that  $x$  must not occur free in  $\psi$  (see Groenendijk and Stokhof 1991 and Brasoveanu 2013 for more discussion):

$$(30) \quad \boxed{(\exists x \varphi); \psi} \Leftrightarrow ([x]; \varphi); \psi \Leftrightarrow [x]; (\varphi; \psi) \Leftrightarrow \boxed{\exists x(\varphi; \psi)}$$

The only thing that is missing is how to relate semantic values specified in terms of sets of assignment pairs to truth conditions. The definition of truth below specifies that:

$$(31) \quad \text{Truth: } \varphi \text{ is true with respect to } g \text{ (relative to a background model) iff there exists an } h \text{ such that } g \llbracket \varphi \rrbracket h.$$

The definition states that a formula  $\varphi$  will be true relative to an input assignment  $g$  as long as we can find at least one output assignment  $h$  such that the pair  $\langle g, h \rangle$  is in the denotation of  $\varphi$ . The formula in (25), for example, is true with respect to an arbitrary input assignment  $g$  if  $g$  can be extended by  $h$  satisfying the bullet points under (29), i.e., if there is at least one man who has a mustache that is black.

For completeness, let us show the representation of a slightly more involved example of clausal conjuncts, discussed in Section 2.1 and repeated here.

$$(32) \quad \text{Mary saw a}^y \text{ man who had bought a}^z \text{ Harry Potter book and who was reading it}_z.$$

$$(33) \quad \exists y(\text{MAN}(y); \text{SEE}(m, y); \underbrace{\exists z(\text{HP\_BOOK}(z); \text{BUY}(y, z)); \text{READ}(y, z)}}_{\text{syntactic scope of } \exists z})$$

Note that the translation is identical to the one given in FOL, we only substituted dynamic conjunction for static FOL conjunction. However, because of the equivalence in (30), the interpretation is correct. In other words, we can rewrite (33) as (34) without any change in meaning. The formula in (33) is true with respect to an arbitrary input assignment if there is at least one man reading a HP book he had bought and Mary saw that man – the correct interpretation.

$$(34) \quad \exists y(\text{MAN}(y); \text{SEE}(m, y); \exists z(\text{HP\_BOOK}(z); \text{BUY}(y, z); \text{READ}(y, z)))$$

At this point, our logic solves anaphora across clausal conjuncts without making any changes to the way anaphoric pronouns in English are translated into the logical representation language. The very same system can also account for donkey anaphora. The solution relies on the dynamic interpretation of implication:

$$(35) \quad \llbracket \varphi \rightarrow \psi \rrbracket = \{ \langle g, h \rangle : g = h \text{ and for any } k \text{ such that } g \llbracket \varphi \rrbracket k, \text{ there exists a } k' \text{ such that } k \llbracket \psi \rrbracket k' \}$$

The first requirement is that the input and output assignments are identical:  $g = h$ . That is, the input assignment can be modified *inside* the implication but once we are done with interpretation of the whole implication, such changes are erased. This behavior is often summarized in the slogan that dynamic implication is *externally static* but *internally dynamic*. The externally static behavior is implemented to capture the fact that a variable assignment modified inside a conditional does not affect anaphora outside the conditional, e.g.:

$$(36) \quad \text{If a}^x \text{ farmer owns a}^y \text{ donkey, he}_x \text{ beats it}_y. \text{ *It}_y \text{ wants to beat him}_x \text{ back.}$$

The rest of the definition in (35) specifies how the internally dynamic behavior is brought about: any variable assignment that satisfies the antecedent of the conditional  $\varphi$ , i.e., that is a possible output relative to the input assignment  $g$ , can be further modified so as to satisfy the consequent of the conditional  $\psi$ . If we introduce the abbreviations in (37a) and (37b), the restatement of the definition of dynamic implication in (37c) concisely shows both the internal dynamics – part of the range of the antecedent update, namely  $g \llbracket \varphi \rrbracket$ , is related to the domain of the consequent update  $\mathbf{dom}(\llbracket \psi \rrbracket)$  – and its universal flavor (set inclusion  $\subseteq$ ).

- (37) a.  $g \llbracket \varphi \rrbracket := \{h : g \llbracket \varphi \rrbracket h\}$   
 b.  $\mathbf{dom}(\llbracket \psi \rrbracket) := \{g : \text{there exists an } h \text{ such that } g \llbracket \psi \rrbracket h\}$   
 c.  $\llbracket \varphi \rightarrow \psi \rrbracket = \{\langle g, h \rangle : g = h \text{ and } g \llbracket \varphi \rrbracket \subseteq \mathbf{dom}(\llbracket \psi \rrbracket)\}$

The dynamic interpretation of the universal quantifier closely follows dynamic implication: it is externally static and internally dynamic. In fact, it is easy to see that just as dynamic existential quantification can be decomposed into a random assignment update and dynamic conjunction, dynamic universal quantification can be decomposed into a random assignment update and dynamic implication.

$$(38) \quad \llbracket \forall x \varphi \rrbracket = \{\langle g, h \rangle : g = h \text{ and for any } k \text{ such that } k[x]g, \text{ there exists a } k' \text{ such that } k \llbracket \varphi \rrbracket k'\}$$

$$(39) \quad \llbracket \forall x \varphi \rrbracket = \llbracket [x] \rightarrow \varphi \rrbracket$$

The last bit of information we need is the translation schema for *every*, which remains the same as in classical FOL, except the universal quantifier and implication receive a dynamic interpretation:

$$(40) \quad \text{every NP VP} \rightsquigarrow \forall x(\text{NP}(x) \rightarrow \text{VP}(x))$$

As a consequence of this change in their semantics, the crucial equivalence in (41) between  $\exists x$  in the antecedent  $\varphi$  and  $\forall x$  with scope over the entire conditional holds across the board – that is, without the usual caveat in classical FOL that the variable  $x$  should not occur free in the consequent  $\psi$ .

$$(41) \quad \boxed{(\exists x \varphi) \rightarrow \psi} \Leftrightarrow ([x]; \varphi) \rightarrow \psi \Leftrightarrow [x] \rightarrow (\varphi \rightarrow \psi) \Leftrightarrow \boxed{\forall x(\varphi \rightarrow \psi)}$$

We now have everything we need to capture donkey anaphora, (42). The compositionally derived interpretation is shown in (43). But given the equivalence in (41), the compositionally derived interpretation is equivalent to the intended interpretation, provided in (44)

(42) Every<sup>x</sup> farmer who owns a<sup>y</sup> donkey beats it<sub>y</sub>.

$$(43) \quad \forall x((\text{FARMER}(x); \underbrace{\exists y(\text{DONKEY}(y); \text{OWN}(x, y))}_{\text{syntactic scope of } \exists y})) \rightarrow \text{BEAT}(x, y))$$

$$(44) \quad \forall x \forall y((\text{FARMER}(x); \text{DONKEY}(y); \text{OWN}(x, y)) \rightarrow \text{BEAT}(x, y))$$

Both (43) and the equivalent (44) denote a set of pairs of assignments  $g$  and  $h$  that are identical and such that for every assignment  $k$  that extends  $g$  so that

- $k$  differs from  $g$  at most with respect to the values of  $x$  and  $y$
- $k(x)$  is a farmer and  $k(y)$  is a donkey
- $k(x)$  owns  $k(y)$

it also holds that:

- $k(x)$  beats  $k(y)$

Although  $y$  in  $\text{BEAT}(x, y)$  is not in the syntactic scope of  $\exists y$  in (43), the dynamic interpretation of  $\rightarrow$  ensures that variables in the consequent are bound by existentials in the antecedent, and it also ensures that the resulting interpretation is equivalent to a universal quantification. This happens because implication is (internally) dynamic, so variable assignments updated in the antecedent are not discarded, they serve as input assignments for the interpretation of the consequent.

This dynamic semantics system can also capture cross-sentential anaphora, (45). It suffices to assume that sequences of sentences are connected by ‘;’. It can also account for donkey conditionals, (46). In fact, because of the equivalence in (41), (46) is interpreted in the same way as (42).

(45)  $A^x$  squirrel ran by.  $It_x$  was hungry.

(46) If  $a^x$  farmer owns  $a^y$  donkey,  $he_x$  beats  $it_y$ .

## 5 Problems with the simple dynamic view of generalized quantification

### 5.1 Unselective generalized quantification and the proportion problem

Our analysis of donkey conditionals requires every variable assignment satisfying the antecedent to be further extendable in such a way that it can satisfy the consequent. The universal quantificational force can be made explicit by modifying the conditional with an adverb, (47a). Indeed, the meaning of (47a) seems very close, if not identical, to the conditional without the adverb *always*. The adverb expresses what quantificational force the indefinites in its scope should receive (indefinites are ‘chameleonic’ with respect to quantificational force, cf. Heim 1982), and can consequently be paraphrased as (47b). Since the adverb (we assumed) does not select which particular indefinites should receive its quantificational force, it is a case of unselective generalized quantification (cf. Lewis 1975).

(47) a. If  $a^x$  farmer owns  $a^y$  donkey,  $he_x$  always beats  $it_y$ .

b. For every farmer-donkey pair, if the farmer in the pair owns the donkey in the pair, the farmer beats the donkey.

So far, our semantics focused on cases in which conditionals are modified by *always* or the adverbial modification is left implicit (which, we tacitly assumed, is identical to using *always*). But there are other possibilities. For example, *usually* can be used, as in (48), to express a quantificational force over variable assignments parallel to the quantificational force of *most* over individuals:

(48) If  $a^x$  farmer owns  $a^y$  donkey,  $he_x$  usually beats  $it_y$ .

(49) For most farmer-donkey pairs, if the farmer in the pair owns the donkey in the pair, the farmer beats the donkey.

We could capture this varying behavior of conditionals by generalizing our dynamic implication as shown below, where  $\mathbf{Q}$  is any quantificational adverb, not just the universal one. Under this view, quantificational adverbs are assumed to relate sets of variable assignments in the same way that their generalized quantifier counterparts relate sets of individuals: for

example *usually* denotes the same relation as *most*, except over variable assignments instead of individuals. For example, in (47a) **Q** is the universal quantifier and the antecedent and consequent updates end up being related by ‘ $\rightarrow$ ’, as shown in (51).

$$(50) \quad \llbracket \mathbf{Q}(\varphi, \psi) \rrbracket = \{ \langle g, h \rangle : g = h \text{ and } \llbracket \mathbf{Q} \rrbracket(g \llbracket \varphi \rrbracket, \mathbf{dom}(\llbracket \psi \rrbracket)) \}$$

$$(51) \quad \llbracket \textit{always}(\varphi, \psi) \rrbracket = \{ \langle g, h \rangle : g = h \text{ and } \llbracket \textit{always} \rrbracket(g \llbracket \varphi \rrbracket, \mathbf{dom}(\llbracket \psi \rrbracket)) \} \\ = \{ \langle g, h \rangle : g = h \text{ and } g \llbracket \varphi \rrbracket \subseteq \mathbf{dom}(\llbracket \psi \rrbracket) \} \\ = \{ \langle g, h \rangle : g = h \text{ and } g \in \mathbf{dom}(\llbracket \varphi \rightarrow \psi \rrbracket) \}^4$$

The complete analysis would require an extension of the first-order logic backbone of the system so that all quantifiers, including *most* and *usually*, could be accommodated. However, we can already see how the schema in (50) allows us to extend our analysis to (52). In this case, **Q** is the existential quantifier and the antecedent and consequent updates end up being related by ‘;’.

(52) Sometimes, if a farmer owns a donkey, he beats it.

$$(53) \quad \llbracket \textit{sometimes}(\varphi, \psi) \rrbracket = \{ \langle g, h \rangle : g = h \text{ and } \llbracket \textit{sometimes} \rrbracket(g \llbracket \varphi \rrbracket, \mathbf{dom}(\llbracket \psi \rrbracket)) \} \\ = \{ \langle g, h \rangle : g = h \text{ and } g \llbracket \varphi \rrbracket \cap \mathbf{dom}(\llbracket \psi \rrbracket) \neq \emptyset \} \\ = \{ \langle g, h \rangle : g = h \text{ and } g \in \mathbf{dom}(\llbracket \varphi; \psi \rrbracket) \}$$

Could this generalized version of donkey conditionals be applied to donkey anaphora in relative clauses like (54)?

(54) Most<sup>x</sup> farmers who own a<sup>y</sup> donkey beat it<sub>y</sub>.

As Kadmon, (1987) and Heim, (1990) observe (see also references therein), the two types of donkey sentences are different and analyzing relative-clause donkey sentences like conditional donkey sentences runs into a ‘proportion’ problem. Quantifying over most pairs  $\langle x, y \rangle$  such that  $x$  is a farmer and  $y$  is a donkey that  $x$  owns is intuitively correct for donkey conditionals like (48), but not for relative-clause donkey sentences like (54).

For example, imagine a village with 10 farmers, 9 of which own 1 donkey (and that’s it) and 1 of which owns 20 donkeys. The 1-donkey farmers never beat their donkeys, while the 20-donkey farmer beats all his donkeys. Sentence (54) is intuitively false in this scenario, since most farmers (9 out of 10) are such that they don’t beat the donkeys they own. The adverbial / unselective truth conditions of (48), however, are satisfied: out of 29  $\langle x, y \rangle$  pairs that satisfy the restrictor formula, 20 (hence, most) pairs also satisfy the nuclear scope formula.

## 5.2 Weak and strong donkey readings

Another problem for the unselective analysis of generalized quantifiers is that it fails to account for the fact that the same donkey sentence can exhibit two different readings, a strong one and a weak one.

Consider again the following sentence, repeated from above:

(55) Every<sup>x</sup> farmer who owns a<sup>y</sup> donkey beats it<sub>y</sub>.

<sup>4</sup>Because dynamic implication is externally static, we could push this sequence of equivalences further and show that  $\llbracket \textit{always}(\varphi, \psi) \rrbracket = \llbracket \varphi \rightarrow \psi \rrbracket$ . We refrain from doing that in the interest of clearly exhibiting the parallel between the way in which *always* and dynamic implication  $\rightarrow$  are related, as shown in (51), and the way in which *sometimes* and dynamic conjunction ; are related, as shown in (53).

We said that the sentence receives a reading paraphrasable as ‘for every farmer and every donkey, if the farmer owns the donkey, he beats it’. This reading is definitely possible, as argued by Geach, (1962), and it is a commendable trait of dynamic semantics that it can derive it. However, sentence (55) can receive another, weak reading: every farmer beats some donkey that he owns, but not necessarily each and every one of them. Chierchia, (1995, p. 64) provides a context in which the most salient reading is the weak one: imagine that the farmers under discussion are all part of an anger management program and they are encouraged by the psychotherapist in charge to channel their aggressiveness toward their donkeys (should they own any) rather than toward each other. The farmers scrupulously follow the psychotherapist’s advice – in which case we can truthfully assert (55) even if the donkey-owning farmers beat only some of their donkeys.

Furthermore, there are donkey sentences for which the weak reading is the most salient one:<sup>5</sup>

(56) Every<sup>x</sup> person who has a<sup>y</sup> dime will put it<sub>y</sub> in the meter.

(57) Yesterday, every<sup>x</sup> person who had a<sup>y</sup> credit card paid his<sub>x</sub> bill with it<sub>y</sub>.

Thus, both weak and strong readings are available (see also Geurts 2002 for experimental evidence) and this is a problem for our unselective notion of dynamic generalized quantification, which allows only the latter. To see this, consider again the equivalence noted above in (41): because of this, existential quantifiers in the antecedent are obligatorily interpreted as universal quantifiers with scope over the entire conditional. In particular, (56) is to be paraphrased as ‘for every person and every dime, if the person has the dime, s/he puts it in the meter’, which is not the correct, weak reading.

The proportion problem and the availability of weak donkey readings point to the fact that the unselective notion of generalized quantification is empirically inadequate and it should be supplemented with a notion of *selective* quantification that relates two sets of individuals and not two sets of assignments. On one hand, relating sets of individuals solves the proportion problem; on the other hand, we can extract the two sets of individuals based on the restrictor and nuclear scope formulas in such a way that both weak and strong donkey readings are available. We will discuss this solution to both problems in the next section.

## 6 Selective generalized quantification and solutions to proportions and weak / strong donkey readings

The notions of selective dynamic generalized quantification proposed in the literature fall into two broad classes. The first class of notions employs a dynamic framework based on single variable assignments (like Discourse Representation Theory, File Change Semantics and Dynamic Predicate Logic, see Kamp, 1981; Heim, 1982; Groenendijk and Stokhof, 1991; Kamp and Reyle, 1993) and analyzes generalized quantification as internally dynamic and externally static. The main idea is that the restrictor set of individuals is extracted based on the restrictor update, while the nuclear scope set of individuals is extracted based on both the restrictor and the nuclear scope update, so that anaphoric connections between the restrictor and the nuclear scope can be captured.

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<sup>5</sup>(56) is due to Pelletier and Schubert, (1989) and (57) is due to R. Cooper, according to Chierchia, (1995).

The second class of notions employs a dynamic framework based on *sets* of variable assignments (e.g., van den Berg, 1996; Nouwen, 2003; Brasoveanu, 2007) and analyzes generalized quantification as both internally and externally dynamic. The main idea is that the restrictor set of individuals is the maximal set that satisfies the restrictor update, and the nuclear scope set of individuals is the maximal (structured) subset of the restrictor set of individuals that satisfies the nuclear scope update. We will discuss this class of approaches in the next section. In the current section we will focus on the first class of approaches.

The most common way of extending classical dynamic semantics with selective generalized quantification was first suggested in Bäuerle and Egli, (1985), Root, (1986) and Rooth, (1987), and was first formulated in terms related to Dynamic Predicate Logic in Eijck and Vries, (1992) and Chierchia, (1992, 1995). The proposal is also adopted in Heim, (1990) and Kamp and Reyle, (1993). We will briefly present the core of these proposals here.

Up until now, we assumed that quantifiers relate sets of assignments, i.e., we took quantification to be unselective (Lewis 1975). We have already indicated one way to think about this for *always* in (51). To extend this to a quantifier over individuals like *every*, we simply have to add the fact that such quantifiers contribute their own variable, i.e., their own random assignment update:

$$(58) \quad \llbracket \text{every}_x(\varphi, \psi) \rrbracket = \{ \langle g, h \rangle : g = h \text{ and } \llbracket \text{always} \rrbracket(g \llbracket [x]; \varphi \rrbracket, \mathbf{dom}(\llbracket \psi \rrbracket)) \} \\ = \{ \langle g, h \rangle : g = h \text{ and } g \llbracket [x]; \varphi \rrbracket \subseteq \mathbf{dom}(\llbracket \psi \rrbracket) \}$$

But these quantifiers are still fundamentally unselective, indicated in (58) by the fact that we still use  $\llbracket \text{always} \rrbracket$  when we interpret the quantifier. For selective quantification, we need quantifiers to relate sets of individuals and not sets of assignments – while still allowing for anaphoric connections between the restrictor and the nuclear scope. As an intermediate step towards this notion of selective quantification, let’s reformulate the unselective quantification in (58) as shown in (59), which is unselective quantification with conservativity (Barwise and Cooper 1981) explicitly built into it. That is, the nuclear scope set of assignments is not simply obtained based on the nuclear scope update  $\mathbf{dom}(\llbracket \psi \rrbracket)$  but it is obtained based on both the restrictor and the nuclear scope updates  $g \llbracket [x]; \varphi \rrbracket \cap \mathbf{dom}(\llbracket \psi \rrbracket)$ :

$$(59) \quad \llbracket \text{every}_x(\varphi, \psi) \rrbracket = \{ \langle g, h \rangle : g = h \text{ and } \llbracket \text{always} \rrbracket(g \llbracket [x]; \varphi \rrbracket, g \llbracket [x]; \varphi \rrbracket \cap \mathbf{dom}(\llbracket \psi \rrbracket)) \} \\ = \{ \langle g, h \rangle : g = h \text{ and } g \llbracket [x]; \varphi \rrbracket \subseteq (g \llbracket [x]; \varphi \rrbracket \cap \mathbf{dom}(\llbracket \psi \rrbracket)) \}$$

Building conservativity into dynamic quantification enables us to capture anaphoric connections by means of the way we construct the nuclear scope set of assignments, since we now have access to both the restrictor and the nuclear scope updates in there. Since we now take care of the anaphoric dynamics at the level of nuclear-scope set construction, we are free to switch to selective quantification and let our quantifiers simply relate sets of individuals, as shown in (60b). To increase readability, we introduce the abbreviation in (60a), which extracts a set of individuals from a set of assignments  $G$  by projecting  $G$  onto a specific variable  $x$ .

$$(60) \quad \text{a. } \lambda x. G := \{ g(x) : g \in G \} \\ \text{b. } \llbracket \text{every}_x(\varphi, \psi) \rrbracket = \{ \langle g, h \rangle : g = h \text{ and } \llbracket \text{every} \rrbracket(\lambda x. g \llbracket [x]; \varphi \rrbracket, \lambda x. g \llbracket [x]; \varphi \rrbracket \cap \mathbf{dom}(\llbracket \psi \rrbracket)) \} \\ = \{ \langle g, h \rangle : g = h \text{ and } \lambda x. g \llbracket [x]; \varphi \rrbracket \subseteq \lambda x. g \llbracket [x]; \varphi \rrbracket \cap \mathbf{dom}(\llbracket \psi \rrbracket) \} \\ = \{ \langle g, h \rangle : g = h \text{ and } \lambda x. g \llbracket [x]; \varphi \rrbracket \subseteq \lambda x. g \llbracket [x]; \varphi; \psi \rrbracket \}$$

The first two (equivalent) ways of expressing the denotation of selective *every*-quantification in (60b) are strictly parallel to the unselective *every*-quantification in (59). We can already see

that selective quantification solves the proportion problem posed by (54) since selective quantifiers relate sets of farmers rather than sets of assignments or sets of farmer-donkey pairs.

The third and final (and still equivalent) formulation in (60b) prepares the way towards the account of weak vs. strong donkey readings. This concise formulation makes explicit that the restrictor and nuclear scope sets of individuals are extracted in a parallel way, the only difference being that the nuclear scope set of individuals involves the nuclear scope update in addition to all the ingredients used to extract the restrictor set of individuals. Importantly, when we extract the nuclear scope set of individuals  $\lambda x. g[[x]; \varphi; \psi]$ , the restrictor formula  $\varphi$  and the nuclear scope formula  $\psi$  are dynamically conjoined, which ensures that we can capture donkey anaphora.

Equally importantly, we can now choose a different dynamic operator to relate the restrictor  $\varphi$  and the nuclear scope  $\psi$ , namely dynamic implication. As noted by Rooth, (1987), Heim, (1990) and Kanazawa, (1994), dynamic conjunction yields weak donkey readings and dynamic implication yields strong donkey readings.

$$(61) \quad \begin{aligned} \text{a. } & \llbracket \mathbf{Q}_x^{weak}(\varphi, \psi) \rrbracket = \{ \langle g, h \rangle : g = h \text{ and } \llbracket \mathbf{Q} \rrbracket(\lambda x. g[[x]; \varphi], \lambda x. g[[x]; \varphi; \psi]) \} \\ \text{b. } & \llbracket \mathbf{Q}_x^{strong}(\varphi, \psi) \rrbracket = \{ \langle g, h \rangle : g = h \text{ and } \llbracket \mathbf{Q} \rrbracket(\lambda x. g[[x]; \varphi], \lambda x. g[[x]; (\varphi \rightarrow \psi)]) \} \end{aligned}$$

Dynamic conjunction  $\varphi; \psi$  in (61a) yields weak donkey readings because of the equivalence in (30): any indefinite in the restrictor  $\varphi$  will bind variables in the nuclear scope  $\psi$  with existential force. Dynamic implication  $\varphi \rightarrow \psi$  in (61b) yields strong donkey readings because of the equivalence in (41): any indefinite in the restrictor  $\varphi$  will bind variables in the nuclear scope  $\psi$  with universal force.

## 7 Plural discourse reference and multiple donkey anaphora

The family of accounts of weak/strong donkey readings presented in the previous section introduced two novelties over the unselective account: (i) quantification relates sets of individuals, not sets of assignments, and (ii) quantifiers like *every* are ambiguous, and the ambiguity accounts for weak and strong readings of donkey anaphora.

We will now turn to the second family of accounts of quantification and donkey anaphora in dynamic semantics. These accounts are not just notational variants of the accounts in the previous section: they generalize the dynamic turn in Kamp, (1981)/Heim, (1982)/Groenendijk and Stokhof, (1991) that was primarily focused on indefinites / existential quantification to generalized quantifiers. That is, generalized quantification is taken to be not only internally but also externally dynamic, which accounts for the fact that unlike singular anaphora, plural anaphora to generalized quantifiers is usually possible (*Every student / Most students left the party early. They/\*She had a final exam the next day.*)

In addition, this second family of accounts also enables us to account for mixed weak and strong donkey readings, exemplified in (62) (from Brasoveanu 2008).

$$(62) \quad \text{Every}^x \text{ person who buys a}^y \text{ book on amazon.com and has a}^z \text{ credit card uses it}_z \text{ to pay for it}_y.$$

If weak and strong readings of donkey anaphora were due to two possible interpretations of the quantifier *every*, as (61) would have it, we would expect multiple instances of donkey anaphora under a quantifier to all have either a strong or a weak reading. This prediction turns

out to be incorrect: the most salient interpretation of (62) is that for *every* book (strong reading) that any credit-card owner buys on amazon.com, there is *some* credit card (weak reading) that s/he uses to pay for the book. This is, then, a mixed reading, which is problematic for the first group of accounts of strong / weak readings of donkey anaphora. Note also that the credit card can vary from book to book, e.g., someone can use her American Express card to buy mathematical logic books and her Visa to buy science fiction novels, which means that even weak indefinites like  $a^z$  credit card can introduce non-singleton (but non-maximal) sets.

One conceivable way of analyzing (62) is to suggest that singular donkey anaphora involves plural reference, i.e., reference to collections / non-atomic individuals, as proposed in Lappin and Francez, (1994). That is, (multiple) singular donkey anaphora is analyzed as the (multiple) plural anaphora in sentence (63) below, where the two plural pronouns  $them_y$  and  $them_x$  are anaphoric to the plural individuals obtained by summing the domains of the quantifier  $every^y$  girl in his class and of the narrow scope indefinite  $a^x$  gift, respectively.

- (63) Bart bought  $a^x$  gift for  $every^y$  girl in his class and asked  $them_y$  /  $the_y$  girls to wrap  $them_x$  /  $the_x$  gifts.

Under this kind of approach, the mixed-reading donkey sentence in (62) would be analyzed as follows. The strong donkey anaphora to books  $y$  involves the sum individual containing all and only the books bought by a given person  $x$ . At the same time, the weak donkey anaphora to credit cards  $z$  involves a non-maximal sum individual (possibly non-atomic) containing some of the credit cards that the same person  $x$  has. Finally, the nuclear scope of (62) is cumulatively interpreted, i.e., given the maximal sum  $y$  of books and the sum  $z$  of some credit cards, we have: (i) for any atomic individual  $y'$  that is in the collection  $y$ , there is an atom  $z'$  that is in  $z$  and  $z'$  was used to pay for  $y'$ , and also (ii) for any atom  $z'$  in  $z$ , there is an atom  $y'$  in  $y$  such that  $z'$  was used to pay for  $y'$ .

Kanazawa, (2001) observes that such a plural reference approach to weak / strong donkey anaphora incorrectly predicts that the infelicitous sentence in (64) below should be acceptable – at least in a situation in which all donkey-owning farmers have more than one donkey. This is because singular donkey anaphora is guaranteed in such a situation to involve reference to non-atomic individuals, hence to be compatible with collective predicates like *gather*.

- (64) \* $Every^x$  farmer who owns  $a^y$  donkey gathers  $it_y$  around the fire at night.

One way to maintain the plural reference approach and derive the infelicity of (64) is to assume (following a suggestion in Neale 1990) that singular donkey pronouns always distribute over the non-atomic individual they are anaphoric to. For example, the singular pronoun  $it_y$  in (64) contributes a distributivity operator and requires each donkey atom in the maximal sum of  $y$ -donkeys to be gathered around the fire at night. The infelicity of (64) follows from the fact that collective predicates apply only to collections / non-atomic individuals.

But this domain-level (as opposed to discourse-level) distributivity strategy is not able to account for slightly more complicated examples involving singular donkey pronouns and disjunctions. Consider the example below:

- (65)  $Every^x$  boy who gets  $a^y$  new toy shares  $it_y$  for a very short time only or doesn't want to share  $it_y$  at all.

The distributivity strategy faces a problem because the intuitively correct reading for the entire VP disjunction in (65) is strong (for *any* toy, that toy is either shared very briefly or not

shared at all), while the individual readings for the two occurrences of the singular pronoun *it* have to be weak (some toys are shared and some aren't). To derive this reading, it would have to be assumed that the logical form contains only one occurrence of *it*, and that it takes scope over the entire VP disjunction. This would violate the Coordinate Structure Constraint or at the very least require uncommon syntactic operations like ATB movement at the logical form only.

The domain-level distributivity strategy will not help us with the following kind of examples either:

(66) Every<sup>x</sup> boy who bought a<sup>y</sup> gift for a<sup>z</sup> girl in his class asked her<sub>z</sub> deskmate to wrap it<sub>y</sub>.

Sentence (66) contains two instances of strong donkey anaphora: we are considering every gift and every girl. Moreover, the restrictor of the quantification in (66) introduces a dependency between the set of gifts and the set of girls: each gift is correlated with the girl it was bought for. Finally, the nuclear scope retrieves not only the two sets of objects, but also the dependency between (i.e., the structure associated with) them: each gift was wrapped by the deskmate of the girl that the gift was bought for. Thus, we have here donkey anaphora to structure / dependencies in addition to donkey anaphora to objects.

This dependency is semantically – and not pragmatically – encoded. Consider the following situation: suppose that Bart buys two gifts, one for Maggie and the other for Lisa; moreover, the two girls are deskmates. Intuitively, sentence (66) is true if Bart asked Maggie to wrap Lisa's gift and Lisa to wrap Maggie's gift and it is false if Bart asked each girl to wrap her own gift. But if the 'wrapping' relation between gifts and girls were semantically vague and only pragmatically supplied (as it is in sentence (63) above), we would predict sentence (66) to be intuitively true even in the second kind of situation.

For this reason, Brasoveanu, (2008) argues that we need plural discourse reference, i.e., sets of variable assignments of the kind argued for in Elworthy, (1995), van den Berg, (1996), Krifka, (1996), Nouwen, (2003), and Brasoveanu, (2007), in addition to plural reference. The basic idea in all these systems is that rather than treating sentences / formulas as denoting relations between assignments, they should be treated as denoting relations between *sets of assignments*. Analyzing donkey anaphora in terms of plural discourse reference rather than domain-level plural reference avoids issues associated with collective predicates like *gather*, and automatically makes available the dependencies between multiple donkey indefinites in the restrictor when these indefinites are anaphorically retrieved in the nuclear scope of a quantifier.

Having established the need for plural discourse reference, the question arises whether we can do away with plural reference by deriving collections (sets of individuals) from plural discourse reference (sets of assignments). One class of approaches (van den Berg 1996; Nouwen 2003; Asher and Wang 2003) make plural reference dependent on plural discourse reference, i.e., they allow variable assignments to store only atomic individuals. Collections / non-atomic individuals are accessed in discourse only by summing over atomic values stores in sets of assignments. In contrast, approaches like Krifka, (1996) make plural discourse reference dependent on plural reference: Krifka's central notion of parametrized sum individuals (originally proposed in Rooth 1987) associates each atom that is part of a sum individual with a variable assignment that 'parametrizes' / is dependent on that atom.

Both kinds of approaches have difficulties with plural donkey anaphora stemming from their asymmetric treatment of domain-level and discourse-level plurality. The second kind of approaches find it difficult to account for the incompatibility between singular donkey anaphora and collective predicates exemplified in (64) because the discourse-level plurality associated

with strong donkey anaphora requires domain-level plurality, which in turn predicts that the collective predicate *gather* should be felicitous. These approaches also have difficulties with examples of donkey anaphora to structure like (66), in which the order / ‘relative scope’ of the anaphors does not reproduce the order / ‘relative scope’ of the antecedents – because the nested structure of the dependencies stored in parametrized sum individuals predicts that we can anaphorically retrieve the entities stored in the parameterizing assignments only if we first retrieve the collection / sum individual that those assignments actually parametrize.

The first kind of approaches have a different set of problems: they do not capture the fact that plural donkey anaphora behaves in many respects just like singular donkey anaphora. Consider the parallel between the multiple plural donkey sentence in (67) below and the multiple singular sentence in (66) above. Note that the collective predicate *fight (each other)* in (67) is felicitous because, in contrast to example (64), we have domain-level non-atomicity introduced by the plural cardinal indefinite *two<sup>z</sup> boys*.

- (67) Every<sup>x</sup> parent who gives a<sup>y</sup> balloon / three<sup>y</sup> balloons to two<sup>z</sup> boys expects them<sub>z</sub> to end up fighting (each other) for it<sub>y</sub> / them<sub>y</sub>.

Thus, (multiple) plural donkey anaphora provides evidence that natural language interpretation requires both plural discourse reference and plural reference and that these two semantic notions of plurality should be formalized as two independent (yet interacting) meaning components. Finally, allowing for both notions of plurality opens the way to an account of weak / strong plural donkey readings that is parallel to the account of weak / strong singular donkey readings. For example, cardinal indefinites like *two* can be either strong, e.g., *two<sup>z</sup> boys* in (67) above, or weak, e.g., *two<sup>y</sup> dimes* in (68) below – which is a minimal variation on the classical weak donkey sentence in (56) above.

- (68) Every<sup>x</sup> driver who had two<sup>y</sup> dimes put them<sub>y</sub> in the meter.

For a recent discussion of weak/strong donkey readings in a plural discourse-reference framework, and for an account that unifies and improves on the various approaches to donkey anaphora mentioned above, see Champollion, (to appear).

## 8 Uniqueness, maximality and the problem of indistinguishable participants

An alternative approach to donkey anaphora is to maintain classical static (FOL) semantics for indefinites and instead place the additional interpretive burden on pronouns, which are analyzed as covert definite descriptions of some kind.<sup>6</sup> There are a variety of approaches along these lines, which differ from each other and from dynamic approaches in non-trivial respects. The original accounts, which came to be known as E-type approaches to donkey anaphora, were formulated in Evans, (1977, 1980), Parsons, (1978), and Cooper, (1979) and there are clear differences between them and the dynamic frameworks discussed in this chapter. The more recent situation-semantics based E-type accounts in Heim, (1990) and Elbourne, (2001, 2005) improve on the original accounts in various respects, particularly the uniqueness problem – see below, but are increasingly harder to distinguish from dynamic approaches, as Heim,

<sup>6</sup>Still a different account is considered by Dekker, (2012) in which pronouns are treated as functions of variable arity, distinct from both definite descriptions and logical variables.

(1990) notes and Dekker, (1994) discusses in detail. For an in-depth discussion of the empirical phenomena falling under the label of E-type anaphora (including but certainly not limited to donkey anaphora) and of the static and dynamic approaches to these phenomena, see Chapter [INSERT REFERENCE TO RICK NOUWEN'S CHAPTER ON E-TYPE PRONOUNS HERE].

For concreteness, let us consider the E-type analysis of donkey sentences in (69) more closely. This account is outlined in Heim, (1990, p. 170) and exhibits in a very clear and concise way the core of probably the most successful subfamily of E-type approaches: it builds on the PTQ (Montague 1973) based E-type account of donkey anaphora in Parsons, (1978), incorporates crucial insights from Evans, (1977), anticipates the NP-ellipsis E-type account in Elbourne, (2001, 2005), and is designed to address the 'formal link' problem that Kadmon, (1987) raises for the E-type account in Cooper, (1979).

(69) The E-type Logical Form (LF) for sentence (14/55) based on Heim, (1990):

Every<sup>x</sup> [farmer(x) that a<sup>y</sup> donkey(y) own(x, y)] beat(x, it<sub>y</sub> [~~donkey(y) own(x, y)~~])

The donkey pronoun *it<sub>y</sub>* is basically interpreted as a definite article whose covert restrictor is provided by conjoining the restrictor and nuclear scope of its indefinite antecedent. The subscripted variable *y* is semantically inert (unlike in the dynamic accounts discussed above) – it simply serves the function of identifying the indefinite antecedent so that its restrictor and nuclear scope can be copied over. The resulting reading is one that has a uniqueness presupposition, contributed by the definite-article-like interpretation of the donkey pronoun *it*. Oversimplifying for readability purposes,<sup>7</sup> the pronoun is interpreted as in (70) (we subscript terms with their types):

(70)  $it_y \rightsquigarrow \lambda P_{et}. \exists z_e (\forall z'_e ([\text{DONKEY}_{et}(z') \wedge \text{OWN}_{e(et)}(x, z')]) \leftrightarrow z = z') \wedge P(z)$

The resulting reading for the donkey sentence (14/55) is basically that every farmer who owns a donkey beats the unique donkey that s/he owns. This incorrectly requires all farmers in the model to own exactly one donkey. Accommodating the uniqueness presupposition in the restrictor of the universal quantification yields a different but still incorrect reading: every farmer who owns a unique donkey beats that donkey.

As Heim, (1990) points out, the undesirable uniqueness presupposition can be significantly weakened by switching to a situation semantics for the universal quantifier: *every* relates sets of situations, and its restrictor is the set of minimal situations featuring a farmer and a donkey s/he owns. The donkey pronoun in the nuclear scope can now select the unique donkey in that minimal situation: uniqueness is relativized to minimal situations and basically rendered vacuous. Two main observations can be made at this point.

First, while the donkey indefinite receives its classical, static existential interpretation under this account, it would be a categorial mistake to classify the overall approach as classical / static: the interpretation of the universal quantifier is as (internally) dynamic as (51/58), since the minimal situations provided by the restrictor are fed into the nuclear scope formula, the interpretation of which extends / 'builds on' them.

Second, using minimal situations fails to render uniqueness presuppositions vacuous in so-called bishop sentences, the theoretical significance of which was first noticed by Hans Kamp (as reported in Heim 1990). A relative-clause bishop sentence is provided in (71) below:

<sup>7</sup>That is, we interpret definite descriptions the Russellian way and ignore the fact that uniqueness is a presupposition, not part of the at-issue content.

(71) Every<sup>x</sup> bishop who meets another<sup>y</sup> bishop blesses him<sub>y</sub>.

The problem that bishop sentences pose for situation-based E-type accounts has become known as the problem of indistinguishable participants:<sup>8</sup> the restrictor set of the universal quantification in (71) consists of minimal situations featuring two bishops meeting each other, so the uniqueness-presupposing definite description contributed by the pronoun *him<sub>y</sub>* fails to refer. Dynamic accounts do not have a similar problem since the pronoun is indexed with the variable / retrieves a discourse referent introduced by the indefinite antecedent.

Various solutions to this problem have been proposed in the E-type literature. Heim, (1990, p. 157) – see also the account in Elbourne, (2005, chapter 4) – follows an idea originating in Kadmon’s DRT account and derives the intuitive interpretation of bishop sentences by allowing LFs to be indexed with multiple situation variables denoting minimal situations satisfying particular parts of a clause. This, together with a richer inventory of covert definite descriptions that pronouns can access, enables E-type approaches to capture bishop sentences. It is difficult to see, however, how indexing LFs with situation variables and allowing subsequent definite descriptions to retrieve them is ultimately different in nature from dynamic accounts in which LFs are directly indexed with variables over individuals.

A particularly intuitive E-type account is put forth in Ludlow, (1994): the two bishops in (71) are distinguished by the different thematic roles they are assigned, so the covert definite description contributed by the pronoun *him<sub>y</sub>* in (71) requires the bishop to be the patient of the meeting situation mentioned in the restrictor – and that bishop is unique. The other bishop is the agent in the same minimal situation, and although there is another minimal situation involving the same individuals but in which the agent and patient thematic roles are flipped (since the meeting relation is symmetric), that is a distinct minimal situation. As Elbourne, (2005) points out, this E-type account also derives the fact that a ‘theta-role-symmetric’ bishop sentence like (72) is infelicitous: the two individuals have the same theta-role, so the covert definite descriptions contributed by the pronouns *he* and *him* will fail to denote because their uniqueness presupposition fails.

(72) \*If a bishop and a bishop meet, he blesses him.

In contrast, dynamic approaches of the kind we examined in this chapter incorrectly predict that a sentence like (72) is correct: nothing prevents the intended coindexation between the two indefinites in the antecedent of the conditional and the corresponding pronouns in the consequent. We will not explore this further except to remark that while it is true that the dynamic approaches to pronoun resolution we discussed rely on an unexamined notion of syntactic coindexation, there are a variety of dynamic frameworks within which the infelicity of (72) can be naturally captured by leveraging Ludlow’s basic insight, from the referent systems in Vermeulen, (1995) to the recent dynamic framework in Bittner, (2014).

We end this chapter with several remarks about another sub-family of E-type accounts of donkey anaphora in general and weak / strong donkey readings in particular. E-type accounts that address the two readings locate ambiguity either on the main quantifier (Heim, (1990) is an example of this). Alternatively, they locate the distinction on the donkey pronoun. Does, (1993) and Lappin and Francez, (1994) are two of the most detailed accounts of this alternative approach.

Situation-based E-type accounts like Heim, (1990) that locate the semantic heavy-lifting in the main quantifier are very close to the dynamic account summarized in (61) above, so

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<sup>8</sup>For plural bishop sentences and the parallel problem they pose for dynamic frameworks, see Schlenker, (2011).

the empirical problems we raised for that account in section §7, particularly multiple mixed-reading donkey anaphora, apply equally to these E-type accounts. One way to address these empirical problems is to switching to a semantics based on sets of situations, which would result in a system parallel to plural dynamic logics.

E-type accounts that locate weak / strong readings in the donkey pronoun are the most clearly distinct from dynamic approaches in that they retain a classical static semantics for both the donkey indefinite and the main quantifier. Lappin and Francez, (1994) assume a Link-style ontology which includes both atomic individuals and sums of individuals, or i-sums for short. Donkey pronouns are analyzed as functions from individuals to i-sums, e.g., in the classical donkey example (14/55), the pronoun *it* denotes a function  $f$  that maps every donkey-owning farmer  $x$  to some i-sum  $f(x)$  of donkeys that  $x$  owns, i.e., the sum of some subset of the donkeys owned by  $x$ . Strong donkey readings are obtained by placing a maximality constraint on the function  $f$  requiring it to select for each  $x$  in its domain the supremum of its possible values, i.e., in our case, the maximal i-sum of donkeys owned by  $x$ . Weak donkey readings are obtained by suspending the maximality constraint, making  $f$  a choice function of sorts mapping  $x$  to one of the corresponding i-sums of donkeys.

E-type approaches that locate weak/strong readings at the level of donkey pronouns can handle a range of mixed weak and strong donkey sentences, but DP-conjunction donkey sentences of the kind first mentioned in Chierchia, (1995) and exemplified below are problematic for them. Example (73) below is from Chierchia, (1995) and both instances of donkey anaphora in this example are strong: every dog and every cat must be fed. Example (74) is based on an example in Brasoveanu, (2007), and it shows that mixed readings are possible: the sentence asserts that every company who hired Vulcans promoted every Vulcan it hired within two weeks, while there is no company who hired Klingons and promoted *some* (that is, any) Klingon it hired within two weeks – that is, donkey anaphora to *a Vulcan* is strong and the donkey anaphora to *a Klingon* is weak.

(73) Every boy who has a dog and every girl who has a cat must feed it.

(74) Every company who hired a Vulcan but no company who hired a Klingon promoted him within two weeks of hiring.

Example (74) poses a problem for Lappin and Francez, (1994) (and Does 1993) because there is only one pronoun in (74), but two distinct donkey readings. Under this kind of E-type of approaches we would need to covertly ‘reconstruct’ two pronouns, e.g., by Right Node Raising of the entire VP, and we would have to provide independent evidence for such an operation (from intonation for example).

These sentences also pose a problem for the hybrid approach to weak / strong readings proposed in Chierchia, (1995), where the weak reading is derived within a dynamic framework (basically the template in (61a) above) and the strong reading is attributed to an E-type reading of the donkey pronoun. Given that Chierchia, (1995) agrees with the observation that examples like (73) and (74) above involve a single pronoun, this approach is faced with the problem of deriving two kinds of semantic representations on just one instance of a pronoun.

The E-type approach in Lappin and Francez, (1994) and the hybrid approach in Chierchia, (1995) could be extended following a suggestion in Chierchia, (1995, pp. 116-117): the donkey pronoun *him* in (74) could be interpreted as denoting the union of two different functions, a maximal one returning i-sums of Vulcans and a non-maximal one returning i-sums of Klingons. This would obviate the need to reconstruct a second pronoun while still deriving mixed

readings. But this strategy does not work in general because the union of two functions is not necessarily a function: if the same company  $x$  hired both a Vulcan and a Klingon, the first function  $f$  will return the Vulcan as  $f(x)$  and the second function  $f'$  will return the Klingon as  $f'(x)$ , so the result of their union is not function and therefore not a suitable kind of meaning for a donkey pronoun. Other refinements could be imagined; for example, when we take the union of two functions  $f$  and  $f'$ , we require the resulting function to return the sum of  $f(x)$  and  $f'(x)$  for any  $x$  that is in the domain of both  $f$  and  $f'$ .

## 9 Conclusion

In this chapter, we reviewed a set of empirical problems faced by formal semantics frameworks that rely on classical static logic. These phenomena challenge the idea that the natural language notions of quantifier scope, binding, coreference, covariation and their compositional interaction match the corresponding notions and their compositional interaction in classical logic.

Traditionally, the solutions to these empirical problems are taken to fall into two classes: on one hand, dynamic accounts preserve the simple interpretation of pronouns as variables and modifies the structure of semantic evaluation so that the observed interactions between quantifiers, indefinites and anaphors can be compositionally derived; on the other hand, E-type approaches preserve the classical static notion of semantic evaluation and generalize the way pronouns are interpreted by postulating they contribute additional covert syntactic and/or semantic structure. While this dichotomy is true if we focus on the original accounts in Kamp, (1981), Heim, (1982), and Groenendijk and Stokhof, (1991) on one hand and Parsons, (1978), Cooper, (1979), and Lappin and Francez, (1994) on the other, the most successful and sophisticated approaches to donkey anaphora and related phenomena cannot be neatly classified as belonging to only one side of this divide. Most of them recruit a variety of resources in their accounts, from enhancements to the semantic evaluation system (e.g., using partial situations or sets of assignments) to non-trivial divisions of labor between the semantics of the main quantifier, the donkey indefinite, the donkey pronoun and the overall pragmatics of donkey sentences, as we could also see in this chapter.

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