Dynamics of questions

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1 Introduction

There is a fruitful line of work in natural language semantics that enriches the standard Montagovian approach to meaning (Montague, 1970, 1973) by assuming more fine-grained objects for sentence denotations than Montague originally did. Two approaches stand out in this respect: dynamic semantics (cf. Nouwen et al. 2016 for an introduction and overview) and inquisitive semantics (cf. Ciardelli et al. 2013).

In dynamic semantics, a sentence is seen as an instruction of how information state should be modified. In this view, a sentence could be seen as a function that takes an old information state and returns its update with the information that the sentence carries. Alternatively, we could understand a sentence meaning as a relation between two information states. The relation expresses what meaning change the sentence brings about. In inquisitive semantics, a sentence is not a proposition (as the Montagovian tradition would have it), but a *set* thereof. It expresses two aspects of content: informative content, as well as inquisitive one. While the former aspect of content signals what the hearer learns when understanding the sentence, the latter aspect signals what issues the hearer is confronted with by the sentence.

Both approaches currently stand apart as two independent theoretical frameworks. Empirically, there is also little interaction between them as they focus on two different sets of phenomena.

Let me elaborate on the second point. Dynamic semantics predominantly studies how context affects interpretation, most notably, how it affects the interpretation of pronominal anaphora. A classical example discussed in dynamic semantics is in (1), after Heim (1982) and Partee. The first sentence in (1-a) is arguably truth-conditionally identical to the first sentence in (1-b). Yet, the two sentences differ in their effect on context, as only the sentence in (1-a) can support the anaphor *it*. In dynamic semantics, this is explained as a consequence of the fact that the noun phrase *one of the ten marbles* introduces a discourse referent that drives the correct interpretation of the pronoun.

(1) a. One of the ten marbles is not in the bag. It is probably under the sofa.

b. Nine of the ten marbles are in the bag. ??It is probably under the sofa.

The main domain of inquiry of inquisitive semantics, on the other hand, is the interpretation of questions. For example, inquisitive semantics studies what is the informative and inquisitive content of wh-questions like (2-a) or yes-no questions like (2-b).

(2) a. Who went to the party?
b. Did John go to the party?

But dynamic semantics and inquisitive semantics do overlap in their empirical coverage, even when we just focus on their predominant domains of inquiry. It has been known for a while that various phrases in questions can license pronominal anaphora. I will focus here on one case at point: wh-words (see also Groenendijk 1998; van Rooij 1998; Haida 2007).

Consider (3), which shows that the wh-word *who* can license the anaphor *he/they* in the follow-up discourse.[1] In (3-a), this happens when the speaker elaborates one question with a new one (coordination of questions in a discourse). In (3-b), the person B answers a question by referring back to the discourse referent introduced with the wh-word.

(3) a. Who^1 went to the party? And what did he/they$_1$ bring as a present?
b. A: Who^1 won the competition? B: You don't know him$_1$. He$_1$ was an outsider.

In fact, we can build examples parallel to (1), showing that even if questions arguably are truth-conditionally parallel, they might still differ in their effect on context. Such examples are presented in (4).

(4) a. Out of ten marbles, which marble is not in the bag? Is it under the sofa?
b. ??Out of ten marbles, which nine marbles are in the bag? Is it under the sofa?

Data such as these seem straightforward and uncontroversial, yet they are hard to capture in inquisitive semantics or in (most implementations of) dynamic semantics. Inquisitive semantics is of little use since it does not model update in discourse caused by questions. And dynamic semantics mainly studies declarative sentences, ignoring the dynamic role of questions (but see Groenendijk 1998; van Rooij 1998; Haida 2007, 2008 for dynamic analyses of questions; we will come back to these). Still, it is not hard to see how progress could be made: we could develop inquisitive dynamic semantics, which should be able to straightforwardly analyze questions and be able to capture the fact that

[1] Here and throughout I use the common convention of indexing (potential) antecedents with superscripts and anaphoric elements with subscripts (Barwise, 1987). If an anaphoric element carries the same number as an antecedent, this signals that the the element is dependent in its interpretation on the antecedent.

questions can introduce discourse referents. This is what this paper considers.

In the next section, Sect. 2, we will introduce inquisitive semantics. Since our goal is to provide a compositional analysis of phenomena at hand, we will fully focus only on inquisitive compositional semantics (cf. Ciardelli et al. 2017). In Sect. 3, we will shortly discuss a dynamic compositional approach (Compositional Discourse Representation Theory, Muskens 1995, 1996). The compositional inquisitive dynamic semantics is presented in Sect. 4, and its application in Sect. 5.

2 Compositional inquisitive semantics

2.1 Background notions in inquisitive semantics

Consider a sentence such as *John walks.* In a classical approach, we would take this to denote the corresponding proposition, i.e., the characteristic function indicating all the worlds in which John walks, or, equivalently, the set of worlds in which John walks.

In contrast to that, inquisitive semantics models a sentence meaning as a persistent (downward-closed) *set* of propositions. The same example would be interpreted not just as the set of worlds in which John walks, but as the family of subsets of the proposition John walks. Obviously, this object is richer than the sentence meaning assigned in the traditional approach. The richness is exploited in inquisitive semantics to represent two kinds of contents that a sentence carries, informative content (which the traditional approach also captures) and inquisitive content (which is not captured in the traditional approach) (cf. Ciardelli et al. 2013, Ciardelli et al. 2015).

The informative content of $\boldsymbol{\varphi}$ states that the actual world is among the worlds that are present in one or another proposition in $[\![\boldsymbol{\varphi}]\!]$. Let us notate the informative content of $\boldsymbol{\varphi}$ as $\textsc{info}(\varphi)$. Then, the following holds:

(5) $\textsc{info}(\varphi) := \bigcup([\![\boldsymbol{\varphi}]\!])$

$\boldsymbol{\varphi}$ also raises an issue (the inquisitive content). What is an issue? Intuitively, the easiest way to think about an issue is to think about propositions that resolve it. An issue raised by $\boldsymbol{\varphi}$ is resolved by propositions present in $[\![\boldsymbol{\varphi}]\!]$. It is natural that if a proposition resolves some issue I, then every stronger proposition also resolves that issue. Therefore, issues are downward-closed: if $[\![\boldsymbol{\varphi}]\!]$ contains a proposition p it also contains all p', $p' \subseteq p$. Furthermore, it is assumed that the inconsistent proposition, $\emptyset$, resolves any issue. Thus, $[\![\boldsymbol{\varphi}]\!]$ always contains $\emptyset$ and is therefore always non-empty. In sum, a sentence meaning is a set of propositions that resolve the issue raised by φ, and such a meaning is downward-closed and non-empty.

Two types of issues can be distinguished. An issue can be trivial or not. If it is trivial, the informative content of $\boldsymbol{\varphi}$ resolves the issue of $\boldsymbol{\varphi}$, that is, $\textsc{info}(\varphi) \in [\![\boldsymbol{\varphi}]\!]$. If this is not the case (i.e., if $\textsc{info}(\varphi) \notin [\![\boldsymbol{\varphi}]\!]$), the issue is non-trivial. In the latter case, we say that the sentence is inquisitive.

Finally, it is going to be useful to single out propositions that contain *precisely enough* information to resolve the issue expressed by $\boldsymbol{\varphi}$. These are the maximal elements of $[\![\boldsymbol{\varphi}]\!]$, called semantic alternatives:

(6) $\textsc{alt}(\varphi) := \{p \in [\![\boldsymbol{\varphi}]\!] |$ there is no $q \in [\![\boldsymbol{\varphi}]\!]$ such that $p \subset q\}$

Note that if $\boldsymbol{\varphi}$ is non-inquisitive, i.e., $\textsc{info}(\varphi) \in [\![\boldsymbol{\varphi}]\!]$, then $\textsc{info}(\varphi)$ is its unique alternative. On the other hand, if $\boldsymbol{\varphi}$ generates multiple alternatives, then $\textsc{info}(\varphi) \notin [\![\boldsymbol{\varphi}]\!]$, which means that $\boldsymbol{\varphi}$ must be inquisitive.

Apart from the inquisitive/non-inquisitive dimension, we can also divide sentences by the informative/non-informative split. Suppose that a sentence appears in the context whose informative content is the set of worlds W. Then the sentence $\boldsymbol{\varphi}$ is informative iff $\textsc{info}(\varphi) \cap W \neq W$.

2.2 Inquisitive semantics and typed lambda calculus

We will now shortly present how compositional inquisitive semantics can be built, using typed lambda calculus.

2.2.1 Preliminaries

Throughout this section, we assume three basic types: e for entities, t for truth values and s for possible worlds. The set of types is the smallest set including basic types and such that, if σ, τ is in the set of types, then $\langle \sigma, \tau \rangle$ is also in the set. We furthermore assume that for any type τ, the logic includes a countable set of τ-constants (written as Con_τ) and a countably infinite set of τ-variables (Var_τ). Given that, we assume that the set of terms of type τ (Term_τ) is defined as:

(7) For any type τ, the set Term_τ is the smallest set such that:
   a. $\mathrm{Con}_\tau \cup \mathrm{Var}_\tau$ are in the set of Term_τ.
   b. $\alpha(\beta)$ is in the set of Term_τ if $\alpha \in \mathrm{Term}_{\langle \sigma, \tau \rangle}$ and $\beta \in \mathrm{Term}_\sigma$.
   c. $\lambda v.\alpha$ is in the set of Term_τ if $\tau = \langle \sigma, \rho \rangle$ and $v \in \mathrm{Var}_\sigma$ and $\alpha \in \mathrm{Term}_\rho$.
   d. $\alpha = \beta$ is in the set of Term_τ if $\tau = t$ and $\alpha, \beta \in \mathrm{Term}_\sigma$ for any type σ.

The function $[\![\quad]\!]_{M,g}$ assigns interpretation to terms. The interpretation of the object $\boldsymbol{\alpha}$ of type τ is an object $[\![\boldsymbol{\alpha}]\!]_{M,g}$ in the corresponding domain D_τ, where D_e, D_t, D_s are the domain of entities, truth values and possible worlds, respectively, and $D_{\langle \sigma, \tau \rangle} = \{f | f : D_\sigma \to D_\tau\}$, the domain of functions from σ to τ. The interpretation of function application and lambda abstraction, as well as '=' is given in (8).[2] First-order logical constants, $\wedge, \vee, \to, \exists, \forall$, can also be defined in this language (cf. van Benthem 1995) and will be used throughout.

(8) a. $[\![\boldsymbol{\lambda v.\alpha}]\!]_{M,g}$ = function $f \in D_\tau^{D_\sigma}$ such that for all $d \in D_\sigma$ it holds

[2] **boldface** marks object language and standard font marks meta-language.

that $f(d) = [\![\boldsymbol{\alpha}]\!]_{M,g[\boldsymbol{v}/d]}$ if $\boldsymbol{v} \in \text{Var}_\sigma$ and $\boldsymbol{\alpha} \in \text{Term}_\tau$. In the meta-language, we will also notate this function as $\lambda v.[\![\boldsymbol{\alpha}]\!]_{M,g[\boldsymbol{v}/d]}$.

b. $[\![\boldsymbol{\alpha}(\boldsymbol{\beta})]\!]_{M,g} = [\![\boldsymbol{\alpha}]\!]_{M,g}([\![\boldsymbol{\beta}]\!]_{M,g})$.

c. $[\![\boldsymbol{\alpha} = \boldsymbol{\beta}]\!]_{M,g} \;\; = T$ if $[\![\boldsymbol{\alpha}]\!]_{M,g} = [\![\boldsymbol{\beta}]\!]_{M,g}$
$= F$ otherwise

2.2.2 Operators and quantifiers in inquisitive semantics

As noted above, the standard Montagovian approach assigns propositions (sets of worlds) to sentences, while inquisitive semantics treats sentences as sets of propositions. Let us notate for some sentence $\boldsymbol{\varphi}$ this classical interpretation as $|\varphi|$ in the meta-language. We use an abbreviation for a family of subsets of $\boldsymbol{\varphi}$, $/ /$:

(9) $/\varphi/ := \lambda p_{\langle s,t\rangle}.p \subseteq |\varphi|$

For example, the sentence *John walks* would receive the interpretation (10-a) in the traditional apprach (a proposition John walks) and the interpretation (10-b) in inquisitive semantics.[3]

(10) a. $[\![$**John walks**$]\!]$= |John walks| (Standard approach)

b. $[\![$**John walks**$]\!]$ = /John walks/
$= \lambda p_{\langle s,t\rangle}.p \subseteq |\text{John walks}|$ (Inq. semantics)

We will now turn to the definition of conjunction and disjunction in inquisitive semantics. We start by defining polymorphic boolean operators, $\sqcap$ and $\sqcup$, generalizing propositional conjunction and disjunction to arbitrary types ending in t.

(11) a. $\sqcap_{\tau\langle\tau\tau\rangle} = \;\; \wedge$ if $\tau = t$ or
$\lambda X_\tau \lambda Y_\tau \lambda Z_{\sigma_1}.X(Z) \sqcap Y(Z)$ if $\tau = \sigma_1\sigma_2$

b. $\sqcup_{\tau\langle\tau\tau\rangle} = \;\; \vee$ if $\tau = t$ or
$\lambda X_\tau \lambda Y_\tau \lambda Z_{\sigma_1}.X(Z) \sqcup Y(Z)$ if $\tau = \sigma_1\sigma_2$

Polymorphic conjunctions and disjunctions can be deployed in inquisitive semantics if we are to express how objects of type $\langle\langle st\rangle t\rangle$ are coordinated. This yields the following interpretation for the conjunction and disjunction of sentences $\boldsymbol{\varphi}$ and $\boldsymbol{\varphi'}$:

(12) a. $\varphi \sqcap \varphi' = \lambda p.\varphi(p) \wedge \varphi'(p)$

b. $\varphi \sqcup \varphi' = \lambda p.\varphi(p) \vee \varphi'(p)$

Note that the deceivingly parallel (12-a) and (12-b) shows a contrasting behavior when we consider inquisitiveness of coordinated expressions. For example, whenever $\boldsymbol{\varphi}$ and $\boldsymbol{\varphi'}$ are non-inquisitive and $[\![\boldsymbol{\varphi}]\!]$ is not a subset of $[\![\boldsymbol{\varphi'}]\!]$ or vice versa, then (12-b) is inquisitive, but (12-a) is not. That is, (12-b) generates two alternatives, $|\varphi|$ and $|\varphi'|$, but (12-a) has only one alternative, $|\varphi| \cap |\varphi'|$.

[3] M and g subscripts are omitted from now on.

Negation of $\boldsymbol{\varphi}$ is the weakest inquisitive meaning φ' whose meet with φ is inconsistent, i.e., such that $\varphi \cap \varphi' = \{\emptyset\}$. We will denote it as $\neg\varphi$. There is a simple recipe to compute it: $\neg\varphi$ amounts to the set of propositions that are incompatible with every element of $[\![\boldsymbol{\varphi}]\!]$.

(13) $\neg\varphi := \lambda p.\forall p' \in \varphi[\neg\exists i[p'(i) \wedge p(i)]]$

The last operation we consider is inquisitive implication, symbolized with $\twoheadrightarrow$. $\varphi \twoheadrightarrow \varphi'$ carves out the set of propositions p such that for every $p' \subseteq p$, if p' is in the set of φ then p' is in the set of φ', as well.

(14) $\varphi \twoheadrightarrow \varphi' := \lambda p.\forall p' \subseteq p[p' \in \varphi \rightarrow p' \in \varphi']$

Finally, we consider existential and universal quantifiers. To highlight the connection with conjunction and disjunction, we start by defining quantifiers polymorphic on their second argument:

(15) a. $\bar{\exists}_{e\tau} =$ $\exists$ if $\tau = t$ or
$\lambda x_e \lambda Y_\tau \lambda Z_{\sigma 1}.\bar{\exists}x(Y(Z))$ if $\tau = \sigma_1\sigma_2$

b. $\forall\!\!\!\forall_{e\tau} =$ $\forall$ if $\tau = t$ or
$\lambda x_e \lambda Y_\tau \lambda Z_{\sigma 1}.\forall\!\!\!\forall x(Y(Z))$ if $\tau = \sigma_1\sigma_2$

The two quantifiers, $\bar{\exists}$ and $\forall\!\!\!\forall$, can now combine with sentences of inquisitive semantics ($\langle\langle st\rangle t\rangle$):

(16) a. $\bar{\exists}x\varphi = \lambda p.\exists x[\varphi(p)]$
b. $\forall\!\!\!\forall x\varphi = \lambda p.\forall x[\varphi(p)]$

2.2.3 A compositional inquisitive semantics fragment

We now specify a small fragment of compositional inquisitive semantics for English.

In doing so, we will make use of the following insight. Sentences are sets of propositions, type $\langle\langle st\rangle t\rangle$. Let us abbreviate this type as T. Given that, we can reverse-engineer types for other elements in a sentence: 1-place predicates should be of type $\langle eT\rangle$, 1-place quantifiers should be of type $\langle\langle eT\rangle T\rangle$, 2-place quantifiers (determiners) should be of type $\langle\langle eT\rangle\langle\langle eT\rangle T\rangle\rangle$ etc.

Based on this insight and using the abbreviations introduced in the previous subsection, we provide lexical meanings for a small fragment of English in (17) to (21).[4]

(17) Verbs and nouns

a. $[\![\textbf{walk}]\!] = \lambda x./Wx/$
b. $[\![\textbf{man}]\!] = \lambda x./Mx/$
c. $[\![\textbf{see}]\!] = \lambda Q\lambda x.Q(\lambda y./S(x,y)/)$

[4]We assume that our logical language contains predicate symbols corresponding to the verbs and nouns in the fragment of English that we are considering (e.g., W stands for walks, j stands for John etc.).

(18) Proper names
$\llbracket \textbf{john} \rrbracket = j$

(19) Connectives

a. $\llbracket \textbf{or} \rrbracket = \lambda X \lambda Y . X \sqcup Y$
b. $\llbracket \textbf{and} \rrbracket = \lambda X \lambda Y . X \sqcap Y$
c. $\llbracket \textbf{not}_{TT} \rrbracket = \lambda q . \neg q$ (Note: we only define propositional negation in this fragment)

(20) 1-place quantifiers

a. $\llbracket \textbf{somebody} \rrbracket = \lambda P_{eT} . \overline{\exists} x (Px)$
b. $\llbracket \textbf{who} \rrbracket = \lambda P_{eT} . \overline{\exists} x (Px)$
c. $\llbracket \textbf{nobody} \rrbracket = \lambda P_{eT} . \neg \overline{\exists} x (Px)$
d. $\llbracket \textbf{everyone} \rrbracket = \lambda P_{eT} . \forall\!\!\!\forall x [Px]$

(21) 2-place quantifiers

a. $\llbracket \textbf{some} \rrbracket = \lambda P_{eT} \lambda Q_{eT} . \overline{\exists} x (Px \sqcap Qx)$
b. $\llbracket \textbf{no} \rrbracket = \lambda P_{eT} \lambda Q_{eT} . \neg \overline{\exists} x (Px \sqcap Qx)$
c. $\llbracket \textbf{everyone} \rrbracket = \lambda P_{eT} \lambda Q_{eT} . \forall\!\!\!\forall x [Px \twoheadrightarrow Qx]$

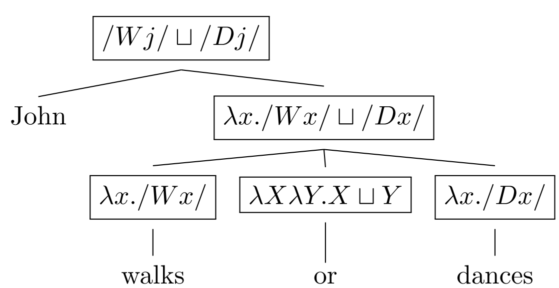
2.3 Examples

We will consider a few examples. More can be seen in Ciardelli et al. (2017).

(22) John walks or dances.

The tree annotated with interpretations is provided in (23).

(23)



Note that the top node is an abbreviation. Expanding it gives us:

(24) $\lambda p . p \subseteq |Wj| \vee p \subseteq |Dj|$

Suppose that we are considering just four worlds, such that John walks and dances in w_1, he only walks in w_2, he only dances in w_3 and he does neither in w_4. The alternatives of the final interpretation are as shown in the left picture of Fig. 1.

A second example is in (25), whose compositional interpretation is given in (26).

(25) Some man sleeps.

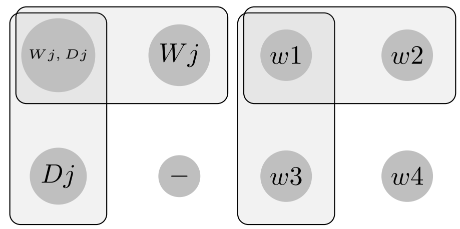
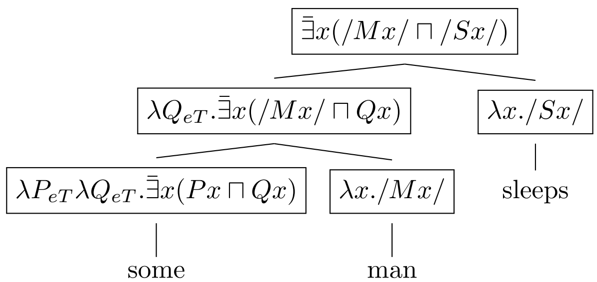


Figure 1: Graphical representation of the alternatives of (24) (left) and the alternatives of (27) (right) in a 4-world context

(26)



Note that the top node is yet again an abbreviation. Expanding it gives us:

(27) $\lambda p.\exists x(p \subseteq |Mx| \wedge p \subseteq |Sx|)$

Suppose that we are considering just four worlds and two men, $m1$ and $m2$. $m1$ sleeps in worlds 1 and 3, $m2$ sleeps in worlds 1 and 2. The alternatives of the final interpretation are as shown in the right picture of Fig. 1.

3 Compositional DRT

3.1 A fragment

Unlike compositional inquisitive semantics, compositional Discourse Representation Theory (CDRT) is well-known. Thus, we will only shortly summarize the main points here. More details can be found in Muskens (1995, 1996). In the presentation, we will slightly diverge from Muskens (1996) (by not considering a special basic type for discourse referents). This move is common for CDRT (cf. Muskens 1995; Brasoveanu 2007; Haida 2007).

CDRT is a type logic with 3 basic types (D_e, entities, D_c, states and D_t, truth values). States are model-theoretic counterparts of variable assignments. Since dynamic semantics treats sentences as relations between assignments, it is clear that CDRT should assign type $\langle c, \langle ct \rangle \rangle$ to sentences. We will abbreviate this type as $\mathbf{t}$. Discourse referents are constant functions from states to entities, $\langle ce \rangle$. This type will be abbreviated as $\mathbf{e}$. Furthermore, we will write discourse referents as u with a subscript and we assume that there is a countably infinite

Type	Abbreviation	Name	Variable
$\langle et\rangle$	-	Static predicates	-
$\langle e\langle et\rangle\rangle$	-	Static relations	-
$\langle c\langle ct\rangle\rangle$	$\mathbf{t}$	Dynamic propositions	r, r'
$\langle\langle ce\rangle\langle c\langle ct\rangle\rangle\rangle$	$\mathbf{et}$	Dynamic predicates	P
$\langle\langle\langle ce\rangle\langle c\langle ct\rangle\rangle\rangle\langle c\langle ct\rangle\rangle\rangle$	$\langle\langle\mathbf{et}\rangle\mathbf{t}\rangle$	Dynamic quantifiers	Q

Table 1: Types, their corresponding names and conventionally used variables in CDRT

set of such constants, i.e., $u_1, \ldots$. Types of other elements are presented in Tab. 1.

When discussing sentences, Muskens (1996) abbreviates their interpretation using the convention from DRT: sentences are represented as DRSs, boxes, which have new discourse referents on the left side and conditions on the right side. For example, $[u_1|C_1]$ would be a DRS which "introduces" one discourse referent, u_1, and consists of one condition, C_1. The abbreviation is defined in (28).

(28) $[u_1, \ldots u_n|C_1, \ldots, C_m] := \lambda k_c \lambda k'_c.k[u_1, \ldots u_n]k' \wedge C_1 k' \wedge \ldots \wedge C_m k'$

Notice that this abbreviation includes another abbreviation, $k[u_1, \ldots u_n]k'$. This statement compares two states, k and k', and requires that k and k' differ at most with respect to $u_1 \ldots u_n$:

(29) $k[\delta_1, \ldots \delta_n]k' := \forall v_{\mathbf{e}}(\delta_1 \neq v \wedge \ldots \wedge \delta_n \neq v \rightarrow v(k) = v(k'))$

Other abbreviations used in CDRT are listed in (30). (Not all of them will be relevant for our fragment.) Even though, generalized dynamic conjunctions and disjunctions will not play a role in the following discussion, we define them here to parallel our presentations with inquisitive semantics. See (30-d) and (30-e). Note that they differ from each other in a non-trivial way. This is because of the way conjunction (merge, ';') and disjunction (or) are treated in (C)DRT. The former one is an operation on DRSs that outputs a new DRS; the latter one is an operation on DRSs that outputs a condition (see also the second definition in (30-b) vs. (30-d)). This contrast will not be important in the rest of the paper.

(30) a. Relations
$R\{\delta_1, \ldots, \delta_n\} := \lambda k_c.R(\delta_1 k), \ldots, \delta_n k))$

b. Operations on DRSs
$\sim$ D $:= \lambda k_c.\neg\exists k'_c(Dkk')$
D or D' $:= \lambda k_c.\exists k'_c(Dkk' \vee D'kk')$
D $\Rightarrow$ D' $:=$ not $(D; [\text{not } D'])$
$:= \lambda k.\forall k'(Dkk' \rightarrow \exists l(D'k'l))$
D $\Leftrightarrow$ D' $:= \lambda k.\forall k'(Dkk' \leftrightarrow D'kk')$

c. Quantifiers
$\exists u_n(D) := [u_n|]; D$
$\forall u_n(D) :=$ not $([u_n|]; [\text{not } D])$
$:= \lambda k_c.\forall k'_c(k[u_n]k' \to \exists l_c(Dk'l))$
$:= [u_n|] \Rightarrow D$

d. Merging
$D_1; D_2 := \lambda k_c \lambda k'_c.\exists l_c(D_1kl \wedge D_2lk')$

e. Generalized dynamic conjunction
$\sqcap_{\tau\langle\tau\tau\rangle} =\ ;$ if $\tau = \mathbf{t}$ or
$\lambda X_\tau \lambda Y_\tau \lambda Z_{\sigma_1}.X(Z) \sqcap Y(Z)$ if $\tau = \sigma_1\sigma_2$ and τ ends in $\mathbf{t}$

f. Generalized dynamic disjunction
$\sqcup_{\tau\langle\tau\tau\rangle} = \lambda r \lambda r'.[|r \text{ or } r']$ if $\tau = \mathbf{t}$ or
$\lambda X_\tau \lambda Y_\tau \lambda Z_{\sigma_1}.X(Z) \sqcup Y(Z)$ if $\tau = \sigma_1\sigma_2$ and τ ends in $\mathbf{t}$

We will now turn to our fragment for English. Lexical meanings of various expressions are listed in (31) to (35).

(31) Verbs and nouns

a. $[\![\textbf{walk}]\!] = \lambda v_{\mathbf{e}}.[|W_{et}\{v\}]$
b. $[\![\textbf{man}]\!] = \lambda v_{\mathbf{e}}.[|M_{et}\{v\}]$
c. $[\![\textbf{see}]\!] = \lambda Q_{\mathbf{ett}} \lambda v_{\mathbf{e}}.Q(\lambda v'_{\mathbf{e}}.[|S_{eet}\{v, v'\}])$

(32) Pronouns

a. $[\![\textbf{he}_n]\!] = \lambda P_{\mathbf{et}}.P(u_n)$

(33) Connectives

a. $[\![\textbf{or}]\!] = \sqcup$
b. $[\![\textbf{and}]\!] = \sqcap$
c. $[\![\textbf{not}]\!] = \lambda r.[| \sim r]$ (Note: we only define propositional negation in this fragment)

(34) 1-place quantifiers

a. $[\![\textbf{somebody}^n]\!] = \lambda P_{\mathbf{et}}.\exists u_n(P(u_n))$
b. $[\![\textbf{who}^n]\!] = \lambda P_{\mathbf{et}}.\exists u_n(P(u_n))$
c. $[\![\textbf{nobody}^n]\!] = \lambda P_{\mathbf{et}}.[| \sim \exists u_n(P(u_n))]$
d. $[\![\textbf{everybody}^n]\!] = \lambda P_{\mathbf{et}}.[|\forall u_n(P(u_n))]$

(35) 2-place quantifiers

a. $[\![\textbf{some}^n]\!] = \lambda P_{\mathbf{et}} \lambda Q_{\mathbf{et}}.\exists u_n(P(u_n); Q(u_n))$
b. $[\![\textbf{no}^n]\!] = \lambda P_{\mathbf{et}} \lambda Q_{\mathbf{et}}.[| \sim \exists u_n(P(u_n); Q(u_n))]$
c. $[\![\textbf{every}^n]\!] = \lambda P_{\mathbf{et}} \lambda Q_{\mathbf{et}}.[|\forall u_n(P(u_n) \Rightarrow Q(u_n))]$

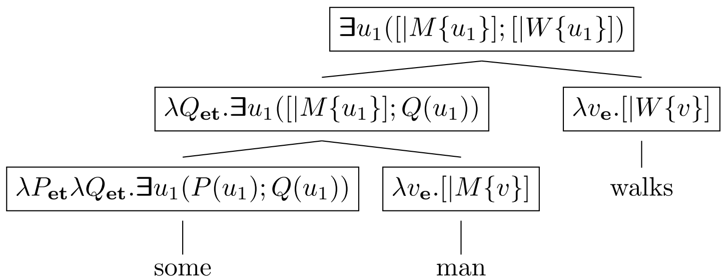
3.2 Examples and discussion

Let us now consider an example:

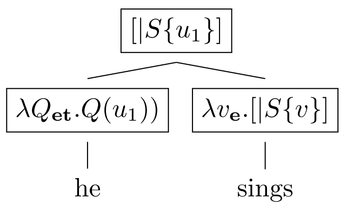
(36) Some^1 man walks. He$_1$ sings.

The compositional interpretation of the first sentence is given in (37) and the compositional interpretation of the second sentence is in (38). The resulting discourse will be created by merging the two sentences, that is (37);(38).

(37)



(38)



The final meaning is:

(39) $\exists u_1([|M\{u_1\}];[|W\{v\}]);[|S\{u_1\}]$

Note that, as was the case in inquisitive semantics, this is only an abbreviation that has to be unpacked. We do so in (40).

(40)
a. $\exists u_1([|M\{u_1\}];[|W\{u_1\}]);[|S\{u_1\}] =$ (by definition of $\exists$)
b. $[u_1|];[|M\{u_1\}];[|W\{u_1\}];[|S\{u_1\}] =$ (by Merging Lemma, Muskens, 1996, p. 150)
c. $[u_1|M\{u_1\},W\{u_1\},S\{u_1\}] =$ (by definition of DRS)
d. $\lambda k\lambda k'.k[u_1]k' \wedge M(u_1(k')) \wedge W(u_1(k')) \wedge S(u_1(k'))$

How is (40-d) interpreted? It is a set of pairs of states (assignments) k and k' such that they differ from each other only wrt u_1 and such that $u_1(k')$ is a man that walks and sings. To sharpen the intuition about the meaning of the sentence, it is helpful to introduce the truth in DRT (cf. Muskens 1996, p. 158):

(41) Truth:
A DRS D (type $\langle c\langle ct\rangle\rangle$) is true with respect to a state k_c iff $\exists k'_c(Dkk')$
A DRS D (type $\langle c\langle ct\rangle\rangle$) is true iff D is true for all states k

Given this, we say that (40-d) is true iff there is a state k' such that $u_1(k')$ is a man that walks and sings. That is, we can write the truth conditions of (40-d) as:

(42) $\forall k\exists k'(k[u_1]k' \wedge M(u_1(k')) \wedge W(u_1(k')) \wedge S(u_1(k')))$

Of course, these are not the standard truth conditions we would consider for (36). However, using the Unselective Binding Lemma (Muskens, 1996, p. 157),

Type (Abbreviated)	Name	Constant	Variable
$\langle st \rangle$	Static propositions		p
$T := \langle\langle st \rangle t \rangle$	Static sentences		
$\langle eT \rangle$	Static predicates	*M(an), W(alks)*	-
$\langle e \langle eT \rangle\rangle$	Static relations	*S(ee)*	-
$\mathbf{t} := \langle c \langle ct \rangle\rangle$	Dynamic propositions	-	-
$\mathbf{T} := \langle\langle st \rangle \mathbf{t} \rangle$	Dynamic inquisitive sentences (Alternatively, inquisitive DRSs)	-	q
$\mathbf{e} := \langle ce \rangle$	Discourse referents	$u_1, \ldots$	v
$\mathbf{eT}$	Dynamic inquisitive predicates	-	P
$\langle\langle \mathbf{eT} \rangle \mathbf{T} \rangle$	Dynamic inquisitive quantifiers	-	Q

Table 2: Types, their corresponding names and conventionally used variables in ICDRT

this is equivalent to the following first-order formula:

(43) $\exists x(M(x) \wedge W(x) \wedge S(x))$

This, indeed, is the interpretation that we would expect (36) to carry. For more details on CDRT, see the papers discussed in this section.

4 Inquisitive CDRT (ICDRT)

4.1 Combining CDRT and compositional inquisitive semantics

Inquisitive semantics assumes a more complex type for sentences than the classical Montagovian approach: they are of type $\langle\langle st \rangle t \rangle$ (abbreviated as T), rather than $\langle st \rangle$. Similarly, dynamic semantics assumes a more complex type for sentences (which are of type $\langle c \langle ct \rangle\rangle$) and names (type $\langle ce \rangle$). Building compositional inquisitive DRT should then proceed by considering such types for sentences and names that keep the complexities of both approaches. A way to proceed is to assume that underlyingly, conditions in a DRS are inquisitive sentences, rather than simple sentences, while discourse referents are of the same type as they were in CDRT. DRSs, then, must be of type that combines T and $\mathbf{t}$. The type considerations doing just that are presented in Tab. 2.

As just said, what we need is an inquisitive DRS whose conditions, however, are not interpreted as in CDRT (functions from states to truth values, or, in other words, functions from states to extensionally interpreted sentences); rather, conditions should be functions from states to T, or, in other words, functions from states to inquisitive sentences. What we just said is captured in our abbreviations on inquisitive DRS and relations (recall that $/\varphi/ := \lambda p.p \subseteq |\varphi|$):

(44) Abbreviations for inquisitive DRS and conditions

a. DRS
$[u_1, \ldots u_n | C_1, \ldots, C_m] := \lambda p \lambda k_c \lambda k'_c . k[u_1, \ldots u_n]k' \wedge C_1(k')(p) \wedge \ldots \wedge C_m(k')(p)$

b. Relations
$R\{\delta_1, \ldots, \delta_n\} := \lambda k_c ./R(\delta_1 k, \ldots, \delta_n k)/$

The remaining abbreviations are given in (45). Their workings should become more apparent when we discuss actual language examples.

(45) a. Operations on DRSs
$\sim \mathrm{D} := \lambda k_c \lambda p . \forall k' \forall p'(\mathcal{D}p'kk' \to \neg \exists i(p(i) \wedge p'(i)))$
$\mathcal{D}$ or $\mathcal{D}' := \lambda k_c \lambda p . \exists k'_c(\mathcal{D}pkk' \vee \mathcal{D}pkk')$
$\mathcal{D} \Rightarrow \mathcal{D}' := \lambda k \lambda p . \forall k' \forall p' \subseteq p(\mathcal{D}p'kk' \to \exists l(\mathcal{D}'p'k'l))$

b. Merging
$\mathcal{D}_1; \mathcal{D}_2 := \lambda p \lambda k_c \lambda k'_c . \exists l_c(\mathcal{D}_1 pkl \wedge \mathcal{D}_2 plk')$

c. Quantifiers
$\boldsymbol{\exists} u_n(\mathcal{D}) := [u_n|]; \mathcal{D}$
$\boldsymbol{\forall} u_n(\mathcal{D}) := ([u_n|] \Rightarrow D)$
$:= \lambda k_c \lambda p . \forall k'_c \forall p' \subseteq p(k[u_n]k' \to \exists l_c(\mathcal{D}p'k'l))$
$:= [u_n|] \Rightarrow D$

d. Generalized dynamic conjunction
$\sqcap_{\tau\langle\tau\tau\rangle} = ;$ if $\tau = \mathbf{T}$ or
$\lambda X_\tau \lambda Y_\tau \lambda Z_{\sigma_1} . X(Z) \sqcap Y(Z)$ if $\tau = \sigma_1\sigma_2$ and τ ends in $\mathbf{T}$

e. Generalized dynamic disjunction
$\sqcup_{\tau\langle\tau\tau\rangle} = \lambda q \lambda q' .[|q \text{ or } q']$ if $\tau = \mathbf{T}$ or
$\lambda X_\tau \lambda Y_\tau \lambda Z_{\sigma_1} . X(Z) \sqcup Y(Z)$ if $\tau = \sigma_1\sigma_2$ and τ ends in $\mathbf{T}$

A fragment of ICDRT is specified only for selected lexical elements, as was the case in previous frameworks. See (46) to (50). Notice that some of these interpretations look identical to CDRT (e.g., (46)). However, they are not identical: the abbreviations used here are different than in CDRT. For example, the expressions in (46-a,b) would be unpacked into dynamic inquisitive predicates, not dynamic predicates.

(46) Verbs and nouns

a. $[\![\textbf{walk}]\!] = \lambda v_{\mathbf{e}} .[|W_{et}\{v\}]$
b. $[\![\textbf{man}]\!] = \lambda v_{\mathbf{e}} .[|M_{et}\{v\}]$
c. $[\![\textbf{see}]\!] = \lambda Q_{\mathbf{eTT}} \lambda v_{\mathbf{e}} . Q(\lambda v'_{\mathbf{e}} .[|S_{eet}\{v, v'\}])$

(47) Pronouns

a. $[\![\textbf{he}_n]\!] = \lambda P_{\langle \mathbf{eT} \rangle} . P(u_n)$

(48) Connectives

a. $[\![\textbf{or}]\!] = \sqcup$
b. $[\![\textbf{and}]\!] = \sqcap$
c. $[\![\textbf{not}]\!] = \lambda q .[| \sim q]$ (Note: we only define propositional negation in this fragment)

(49) 1-place quantifiers

a. $[\![\textbf{somebody}^n]\!] = \lambda P_{\mathbf{eT}}.\exists u_n(P(u_n))$
b. $[\![\textbf{who}^n]\!] = \lambda P_{\mathbf{eT}}.\exists u_n(P(u_n))$
c. $[\![\textbf{nobody}^n]\!] = \lambda P_{\mathbf{eT}}.[| \sim \exists u_n(P(u_n))]$
d. $[\![\textbf{everybody}^n]\!] = \lambda P_{\mathbf{eT}}.[|\forall u_n(P(u_n))]$

(50) 2-place quantifiers

a. $[\![\textbf{some}^n]\!] = \lambda P_{\mathbf{eT}}\lambda Q_{\mathbf{eT}}.\exists u_n(P(u_n); Q(u_n))$
b. $[\![\textbf{no}^n]\!] = \lambda P_{\mathbf{eT}}\lambda Q_{\mathbf{eT}}.[| \sim \exists u_n(P(u_n); Q(u_n))]$
c. $[\![\textbf{every}^n]\!] = \lambda P_{\mathbf{et}}\lambda Q_{\mathbf{et}}.[|\forall u_n(P(u_n) \Rightarrow Q(u_n))]$

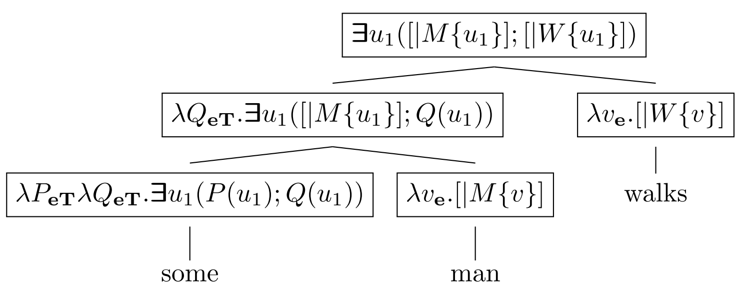
4.2 Examples and discussion

Let us consider the first sentence of the example that we used in the section about CDRT.

(51) Some[1] man walks.

Using abbreviations, the compositional interpretation will look almost identical as the one in Sect. 3.2, see (52).

(52)



Even though this formula looks deceptively similar to the one arrived at in CDRT, it is different, as we can see when we unpack it.

(53)
a. $\exists u_1([|M\{u_1\}]; [|W\{u_1\}])$ = (by definition of $\exists$)
b. $[u_1|]; [|M\{u_1\}]; [|W\{u_1\}]$ = (by Merging Lemma, Muskens, 1996, p. 150)
c. $[u_1|M\{u_1\}, W\{u_1\}]$ = (by definition of inquisitive DRS&relations)
d. $\lambda p\lambda k\lambda k'.k[u_1]k' \wedge /M(u_1(k'))/(p) \wedge /W(u_1(k'))/(p)$ = (by definition of slashes)
e. $\lambda p\lambda k\lambda k'.k[u_1]k' \wedge p \subseteq |M(u_1(k'))| \wedge p \subseteq |W(u_1(k'))|$

How is (53-e) interpreted? It is a set of triples $\langle$proposition, state k, state $k'\rangle$ such that the two states differ from each other only wrt u_1 and such that the proposition is a subset of the proposition $u_1(k')$ being a man and $u_1(k')$ walking.

In general, the meaning of an inquisitive DRS $\mathcal{D}$ specifies two things:

1. given k and k', which propositions support $\lambda p.\mathcal{D}(p)(k)(k')$,[5] and
2. given p, how k can be transformed into k'.

[5] A proposition p supports an inquisitive sentence $\boldsymbol{q}$ iff $p \in [\![\boldsymbol{q}]\!]$.

We will focus on the point 1 and sharpen our understanding of the support. The formula in (52-e) tells us what propositions support the sentence relative to k and k'. But generally, we are interested in support irrespective of what input state k we select. Let us see how we can extract that information.

First, let us note that we can think of (53-e) as a function from propositions to (non-inquisitive) DRSs. Furthermore, we know how to understand truth conditions that a DRS specifies. The definition of truth is repeated here:

(54) Truth:
A DRS D (type $\langle c\langle ct\rangle\rangle$) is true with respect to a state k_c iff $\exists k'_c(Dkk')$
A DRS D (type $\langle c\langle ct\rangle\rangle$) is true iff D is true for all state k

Which propositions support (53-e) irrespective of k? The answer to that is: the propositions $\boldsymbol{p}$ for which $[\![\textbf{(53-e)}(\boldsymbol{p})]\!]$ is true. This is the set of (55) (using the definitions of Truth in (54)).

(55) $\{p|\forall k\exists k'(k[u_1]k' \wedge p \subseteq |M(u_1(k'))| \wedge p \subseteq |W(u_1(k'))|)\}$

This can be simplified using the Unselective Binding Lemma as follows:

(56) $\{p|\exists x(p \subseteq |M(x)| \wedge p \subseteq |W(x)|)\}$

This is intuitively the correct inquisitive object. For example, if we used our inquisitive compositional fragment for (51), we would get to exactly the same interpretation for (51). But there is a crucial difference: while inquisitive compositional semantics is static, this fragment is dynamic. We can see the former fact on the discourse in (57). The pronoun *he* would not be bound in inquisitive compositional semantics and we would not get the desired interpretation.

(57) Some^1 man walks. He$_1$ sings.

On the other hand, ICDRT has no problem with such examples. It would assign the interpretation shown in (58-a). If we are interested in propositions that support this inquisitive DRS, we can unpack (58-a) into (58-b), which, clearly, is the correct interpretation.

(58) a. $\lambda p\lambda k\lambda k'.k[u_1]k' \wedge p \subseteq |M(u_1(k'))| \wedge p \subseteq |W(u_1(k'))| \wedge p \subseteq |S(u_1(k'))|$
b. $\{p|\exists x(p \subseteq |M(x)| \wedge p \subseteq |W(x)| \wedge p \subseteq |S(x)|)\}$

5 Predictions of ICDRT

We will start by updating our definitions of *informative* and *inquisitive*, which were defined with static inquisitive semantics in mind and are not sufficient here. It turns out that the only definition we have to modify is that of INFO().

(59) $\text{INFO}(\mathcal{D}) := \bigcup\{p|\forall k\exists k'\mathcal{D}(p)(k)(k')\}$

Now, we will proceed by carving out questions as semantic objects and discussing compositional analyses of question discourses in which pronominal anaphora is dependent on wh-words.

What kind of semantic objects are questions? Obviously, they are inquisitive. Furthermore, following AnderBois (2012), we assume that their truth-conditional content is non-informative. These are necessary, but not sufficient conditions to define a question. It is possible that questions have to satisfy other requirements, e.g., syntactic and intonational ones. For example, English sentences must show an auxiliary inversion and/or wh-movement and/or specific intonational patterns to be considered questions. We will leave these non-semantic issues aside.

Is it so that all questions are non-informative and inquisitive?

(60) $\mathcal{D}$ is informative wrt W iff $\textsc{info}(\{p|p \text{ supports } \mathcal{D}\}) \neq W$.

Consider the following example.

(61) Who is sleeping?

Two perspectives could be taken here. On one hand, we could say that the sentence should be analyzed as (62).

(62) $[|\exists u_1([|S\{u_1\}]) \text{ or } \sim \exists u_1([|S\{u_1\}])]$

Suppose there are two individuals, a and b and they either sleep or not. Then, (62) is the set of triplets $\langle p, k, k\rangle$ such that p is either the proposition that a sleeps, or that b sleeps, or that neither of them sleeps (and all subsets of those). Clearly, the formula is non-informative for this as well as any other context.

On the other hand, we could say that the truth-conditional content of (61) is a simpler inquisitive DRS, (63). In the same scenario, (63) would be the set of triples $\langle p, k, k'\rangle$ s. t. p is either the proposition that a sleeps and $u_1(k') = a$, or the proposition that b sleeps and $u_1(k') = b$ (and all subsets of those). Now, the formula is informative, even in this context, since the informative content excludes no-sleeper worlds.

(63) $[|\exists u_1([|S\{u_1\}])]$

Crucially, though, there is evidence that posing a question such as (61) already presupposes that somebody is sleeping (existential presupposition). For example, the following question-answer pair is infelicitous, which follows if the informative content of *Somebody is sleeping* does not enhance the informative content of the question due to existential presupposition.

(64) Q: Who is sleeping?
A: # Somebody.

Taken existential presupposition into account, we can conclude that (61) would be inquisitive and non-informative even under (63).

From the two options to analyze question, we will select the second strategy, e.g., (63) for (61). We do so for two reasons. First, such an account nicely captures the fact that in many languages wh-words and indefinites are formally identical. This follows because under this account, wh-words and indefinites have the same denotation, and the only difference arises through existential presupposition in questions, which is arguably caused by question focus (cf. Haida 2007; AnderBois 2012). Secondly, as just indicated, this solution explains the awkwardness of (64).

Following this route also allows us to straightforwardly capture dynamic properties of questions.

First, we can explain why wh-words can antecede pronouns. For example, the first question in (65) will receive the interpretation in (66-a), the second question will be interpreted as (66-b).[6]

(65) Who^1 is walking? Is he$_1$ smiling?

(66) a. $\lambda p \lambda k \lambda k'.k[u_1]k' \wedge p \subseteq |W(u_1(k'))|$
b. $[|[|D\{u_1\}]$ or $\sim [|D\{u_1\}]] = \lambda p \lambda k \lambda k'.k[]k' \wedge p \subseteq |S(u_1(k'))| \vee \forall p' \subseteq |S(u_1(k'))|(\neg \exists i(p'(i) \wedge p(i)))$

Together, they will receive the following interpretation:

(67) $\lambda p \lambda k \lambda k'.k[u_1]k' \wedge p \subseteq |W(u_1(k'))| \wedge (p \subseteq |S(u_1(k'))| \vee \forall p' \subseteq |S(u_1(k'))|(\neg \exists i(p'(i) \wedge p(i))))$

Using our observations from the previous section, we note that the following set supports (67):

(68) $\{p | \exists x(p \subseteq |W(x)| \wedge (p \subseteq |S(x)| \vee \forall p' \subseteq |S(x)|(p'(i) \cap p(i) = \emptyset)))\}$

Suppose we have 2 individuals and 6 worlds, such that *a* and *b* smile in w1 – w3 and do not smile in w4 – w6 (Fig. 2). Furthermore, *a* walks in worlds w1, w2, w4 and w5 and *b* walks in w2, w3, w4 and w6. Then, propositions in the set of (68) will be the ones as indicated by the graphical representation of alternatives in Fig. 2. This is good. We derive the correct meaning of the string of questions, more in particular, the dynamic properties of *who* are correctly dealt with. Cases such as these are hard to deal with for Haida (2007). This is because for Haida (2007), building on partition semantics, questions are externally static. To account for (65), and in particular, for binding of *he* by *who* it has to be stipulated that the presuppositional dimension of the wh-question can bind the pronoun.

A slightly more complex example of dynamic properties of wh-words is provided in (69), which shows that wh-words can participate in quantificational subordination. (69-a) illustrates quantificational subordination outside of the realm of wh-questions. The first sentence of (69-a) makes quantificational environment that the interpretation of the pronoun in the second sentence is par-

[6]We assume that yes-no questions are interpreted as $?q$. The question operator **?** is defined as:

(i) $[\![\mathbf{?}]\!] = \lambda q.q \sqcup [| \sim q]$

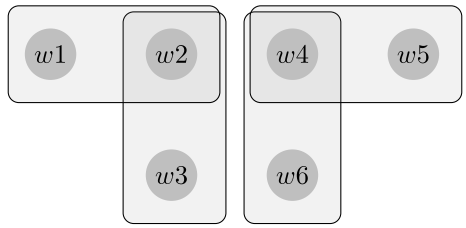


Figure 2: Graphical representation of the alternatives of (65)

asitic upon. Similarly, the interpretation of the pronoun *he* and *it* is parasitic upon the quantificational environment created in the first conjunct of (69-b).

(69) Quantificational subordination
  a. Every player chooses a pawn. He puts it on square one. (Roberts, 1989)
  b. Who brought what cake and how much did he eat of it? (van Rooij, 1998)

We derive this behavior. We show this on the following, simplified example:

(70) Who[1] owns what[2] and whom[3] will he_1 give it_2?

The compositional procedure gives us the following interpretation:

(71) $[u_1, u_2|O\{u_1, u_2\}]; [u_3|G\{u_1, u_2, u_3]$

After unpacking abbreviations and simplifying the formula, we get the following set of propositions supporting (70):

(72) $\{p|\exists x, y(p \subseteq |O(x, y)| \wedge \exists z(p \subseteq |G(x, y, z)|))\}$

This will be resolved by propositions that specify for a person x and a thing y that x owns y and for a possibly different person z that x gave y to z. This seems correct. In particular, notice that the interpretation of the pronoun *it* covaries with *he*, as should be the case in quantificational subordination.

Finally, donkey-style anaphora, possible with wh-words, can also be derived. In particular, (73-b), in which *es* 'it' covaries with *was* 'what' is analyzed correctly.

(73) Donkey-style anaphora
  a. Every farmer who owns a donkey beats it.
  b. Wer kaufte was und verschenkte es sofort?
     who bought what and gave-away it immediately
     literally: 'Who bought what_i and gave it_i away immediately?' (Haida, 2008)

Haida, who discusses such cases, focuses on German, arguing that similar examples are not possible in English. But according to one native speaker I consulted, donkey-style anaphora are grammatical in English. Assuming that the native speaker happens to be correct, we will simplify our position by focusing on a simple English example, (74-a), which receives the interpretation in (74-b) and can be simplified as (74-c).

(74) a. Who^1 owns what^2 and likes it$_2$?
b. $[u_1, u_2|O\{u_1, u_2\}]; [|L\{u_1, u_2\}]$
c. $[u_1, u_2|O\{u_1, u_2\}, L\{u_1, u_2\}]$

The following set of propositions support the formulas in (74-b,c).

(75) $\{p|\exists x, y(p \subseteq |O(x, y)| \wedge p \subseteq |L(x, y)|)\}$

This is correct. The formula is resolved by any proposition that established for a person x and thing y that x owns and likes y.

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